

Test III Review Document

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7.3: As indicated on Test II, there will be a question on 7.3 on this exam, quite similar to the question on the last.

In both of the following parts, state the correct trigonometric substitution. Carry out the integration in one of the parts.

1.

$$\int \frac{1}{\sqrt{x^2 + 1}} dx$$

2.

$$\int \sqrt{4 - x^2} dx$$

7.4: Integration by partial fractions will appear. Problems will be reasonably simple.

Determine the antiderivatives (there are more examples here than would appear as parts in the test question).

1.

$$\int \frac{2x - 1}{x^2 - 5x + 6} dx$$

2.

$$\int \frac{x + 1}{x^2(x + 2)} dx$$

3.

$$\int \frac{2x - 1}{x(x^2 + 1)} dx$$

4.

$$\int \frac{x+1}{x^2-2x+2} dx$$

Hint: complete the square and do an appropriate substitution.

7.6: Computation of numerical estimates with few intervals by the trapezoidal rule and/or Simpson's rule. Estimation of the number of intervals required to estimate an integral using one of these rules to given accuracy, with the error estimate formulas given on your paper.

Estimate

$$\int_0^3 \frac{10}{x^3} dx$$

using six intervals using Simpson's Rule. Use as many decimal places of accuracy as your calculator provides.

How many partitions would you need to get an estimate of this integral with error less than .001 using Simpson's Rule (on the test, the relevant formula would be given here on your paper).

7.7: Compute improper integrals as limits. Expect to be able to answer some question about convergence or divergence of improper integrals by comparison.

1. Compute

$$\int_0^{\infty} xe^{-x} dx$$

Show the setup as a limit and complete the calculation.

2. Compute

$$\int_0^1 \frac{1}{\sqrt[3]{x}} dx$$

Show the setup as a limit and complete the calculation.

3. Determine for each integral whether it converges or diverges by comparison with an integral whose convergence or divergence behavior you know. Explain (say what integral you are comparing it with, what the known convergence or divergence behavior of that integral is, and state whether the given integral converges or diverges).

(a)

$$\int_1^{\infty} \frac{1}{x^3 + e^x}$$

(b)

$$\int_1^{\infty} \frac{1}{\sqrt{x} - e^{-x}}$$

8.1: Compute the first few terms of a sequence whose definition is given. The definition might be recursive. Determine the limit of a sequence; this will generally rely on limit methods you already know. You might need to use L'Hôpital's Rule, so be sure you brush up on it.

1. Compute the first five terms of the sequence $a_n = \frac{n^2}{n!}$
2. Compute the first five terms of the sequence defined by $a_1 = 1; a_2 = 2; a_{n+2} = a_n + 2a_{n+1}$.
3. Determine the limits.
 - (a) $\lim_{n \rightarrow \infty} \frac{n+2}{2n-1}$
 - (b) $\lim_{n \rightarrow \infty} \frac{\ln(n)}{n}$
 - (c) $\lim_{n \rightarrow \infty} \frac{n!}{n^{100}}$ (Don't try to use any computational rules on this one, just say what happens).

8.2: Exhibit the first few terms and possibly compute partial sums of a given series. Recognize geometric series and be able to compute their partial sums and the sums of entire infinite geometric series. Be able to recognize when a geometric series converges and when it diverges. Be able to compute values of a completely obvious telescoping series (with the telescoping information given!)

1. Write out the first five terms of $\sum_{n=1}^{\infty} \frac{3}{2^n}$. Compute the fifth partial sum (you may use your calculator, with as many decimal places of accuracy as it provides. Compute the exact limit, showing supporting work.
2. Write a formula for $\sum_{n=1}^{\infty} \frac{3x}{(2x-1)^n}$, and state for what values of x this series converges.
3. Evaluate the limit $\sum_{n=1}^{\infty} [\arctan(n) - \arctan(n-1)]$. You should remember that $\lim_{n \rightarrow \infty} \arctan(n) = \frac{\pi}{2}$, not 0!