In this paper Hurewicz considers sequences of continuous real-valued functions on metrizable spaces $M$. For such a sequence $(f_n : n < \infty)$ of continuous real-valued functions he considers the subset $\{(f_n(p) : n < \infty) : p \in M\}$ of $\mathbb{R}$ of sequences of real numbers. This is called the value set of the sequence of functions. He defines the usual notions of being bounded and of not being a dominating family in the eventual domination order on $\mathbb{R}$.

In section 2 (p. 194) Hurewicz observes:

A) If $M$ is a union of countably many compact subsets, then the value-set of any sequence of continuous real-valued functions on $M$ is bounded.

In section 3 (p. 194) he gives an example that

B) If $M$ is the set of irrational real numbers between 0 and 1 then there is a sequence of continuous functions on it whose value set is a dominating family.

And for each $n$ he defines the function $f_n$'s value at $x$, $f_n(x)$, to be the $n$-th term in the continued fraction expansion of $x$. He observes that each $f_n$ is continuous and that the value set of this sequence is the set of all $\omega$- sequences of natural numbers. Then he states that from these observations the question arises: What internal properties of a set $M$ are characterized by the fact that the value set of any sequence of real-valued continuous functions on it is bounded (respectively not dominating)?

In section 4 (p. 195-6) he introduces covering properties $E^*$ and $E^{**}$.

In the current SPM notation, $E^* = S_{\text{fin}}(O, O)$ and $E^{**} = U_{\text{fin}}(O, \Gamma)$, where $O$ denotes the collection of open covers of a space, and $\Gamma$ denotes the collection of open $\gamma$ covers. Thus $E^*$ is also what is called the Menger property, and $E^{**}$ is what is called the Hurewicz property. Then Hurewicz proves

C) $S_{\text{fin}}(O, O) = S_{\text{fin}}(\Lambda, \Lambda)$

and he observes that

D) $S_{\text{fin}}(O, O)$ is preserved by continuous images (Footnote 2, page 195).

In footnote 1 on p. 196 Hurewicz states that evidently $E^{**}$ implies $E^*$, and notes that it is an open problem whether there is a set with property $E^*$ but not property $E^{**}$. Then an additional remark is added during the corrections stage of the paper (“Zusatz bei der Korrektur”): In the remark Hurewicz proves:

E) If $M$ is a subset of a separable metric space and has property $E^{**}$ but empty interior in the space, then $M$ is of first category.

This might be due to Sierpinski, on account of the following. Hurewicz states that Sierpinski observed that if the Continuum Hypothesis (CH) holds then the open problem has a positive answer. The argument is as follows: The Lusin set has property $E^*$, but because of $E$, does not have property $E^{**}$.

F) A Lusin set has property $S_{\text{fin}}(O, O)$ but not property $U_{\text{fin}}(O, \Gamma)$.

In section 5 (p. 196 - 199) Hurewicz proves the following theorem:
G) For a separable metric space $M$ the following are equivalent:

1. $M$ has property $S_{\text{fin}}(O, O)$ (respectively $U_{\text{fin}}(O, \Gamma)$).
2. Every sequence of continuous real-valued functions on $M$ has a non-dominating (respectively bounded) value set.

and then adds

H) For a separable metric space $M$ the following are equivalent:

1. $M$ has property $S_{\text{fin}}(O, O)$.
2. For any sequence $\left(f_n : n < \infty \right)$ of continuous real-valued functions on $M$, if the sequence $a$ of real numbers does not belong to the value set of $\left(f_n : n < \infty \right)$, then there are sequences $\beta$ and $\gamma$ of real numbers such that
   a) $\beta < a < \gamma$ and
   b) No $\xi$ in the value set of $\left(f_n : n < \infty \right)$ satisfies $\beta < \xi < \gamma$.

(For this property Hurewicz uses the terminology “the value set of $\left(f_n : n < \infty \right)$ is closed”.)

In Section 6 (p. 199 - 200) Hurewicz discusses Menger’s Conjecture that in metrizable spaces $S_{\text{fin}}(O, O)$ implies $\sigma$-compactness. He reminds the reader that in his 1925 paper he proved that this conjecture is true for analytic sets, and then remarks that the characterization in G) above characterizes the $\sigma$-compact sets among the analytic sets as those on which the value sets of a sequence of continuous real-valued functions is bounded. He also observes that Sierpinski showed that the Continuum Hypothesis implies the negation of Menger’s Conjecture. Then, on page 200 Hurewicz conjectures that $U_{\text{fin}}(O, \Gamma)$ is equivalent to $\sigma$-compactness.

In Section 7 (p. 201 - 202) Hurewicz connects this study with problems of Hausdorff—whether there is an unbounded sequence of ordertype $\omega_1$ in the eventual domination order on sequences of reals, or such a dominating sequence. Though Hurewicz considers the first uncountable cardinal number, his next result adapts directly to the modern dominating number and bounding number:

I) The minimal cardinality of an unbounded set in $\mathbb{R}$ is the same as the minimal cardinality of a separable metric space without property $U_{\text{fin}}(O, \Gamma)$.

and he states that quite analogously one proves

J) The minimal cardinality of a dominating set in $\mathbb{R}$ is the same as the minimal cardinality of a separable metric space without property $S_{\text{fin}}(O, O)$.

Supplementary Section (p. 202 - 204) In this supplementary section Hurewicz proves:

K) For a metrizable space $M$ the following are equivalent:

1. $M$ has property $U_{\text{fin}}(O, \Gamma)$.
2. Each metrizable continuous image of $M$ is a union of countably many totally bounded sets.

Thus, a subspace of a metrizable space has property $U_{\text{fin}}(O, \Gamma)$ if, and only if, it is sigma-totally bounded. He then reformulates the question behind his conjecture. If a metrizable space has property (2) of K, must it be $\sigma$-compact? And in the footnote on page 204 Hurewicz notes that similarly:

L) For a metrizable space $M$ the following are equivalent:

1. $M$ has property $U_{\text{fin}}(O, O)$. 

(2) Each metrizable continuous image of M is a union of countably many sets whose diameters converge to zero.

Remarks:
1) Notation: Menger used the symbol E to denote a basis property which he introduced in his 1924 paper. In his 1925 paper where Hurewicz studied Menger’s basis property, he used the symbol E* to denote a covering equivalent to Menger’s basis property. In the current SPM notation, the property E* is denoted $S_{fin}(O, O)$ where O denotes the collection of open covers of a space. And Hurewicz introduced in the 1925 paper a second covering property which was denoted by the symbol E**. In the current SPM notation, property E** is denoted $S_{fin}(\Omega, O^{gp})$.
2) Hurewicz was familiar with the 1909 papers by Hausdorff on the eventual domination order, and with Hausdorff’s 1917 monograph “Set Theory”.