MATH 497 / 597 – Homework 7

Your work on this assignment must be handed in by Tuesday, 26 November 2002 at 3:15 p.m. GOOD LUCK!

1) Consider the values x_j, $j = 0, 1, \ldots, m$. These values define the "hat" or "chapeau" functions $L_j(x)$, $j = 0, 1, \ldots, m$, that we used in the one-dimensional Galerkin method. Prove:

$$\sum_{j=0}^{m} L_j(x) = 1$$

for all $x \in [x_0, x_m]$.

2) Along with the definitions in Exercise 1) above, consider the vector space $\mathcal{P}$ of piecewise linear polynomials on $[x_0, x_m]$, where each "piece" is defined on the on the interval $[x_{j-1}, x_j]$, $j = 1, 2, \ldots, m$. Prove that the functions $L_j(x)$, $j = 0, 1, \ldots, m$ are a *basis* (in the usual linear algebra sense) for $\mathcal{P}$.

3) Write code to implement the one-dimensional Galerkin method for the differential equation

$$\frac{d^2u}{dx^2} = f(x) \tag{1}$$

on the interval $[0, 1]$ with Dirichlet boundary conditions. Use Simpson's method to evaluate the integrals involving the function f.

4) **Required only for graduate students.** Consider Galerkin's method for solving (1) with boundary conditions $u(0) = \alpha$ and $\frac{du}{dx}(1) = \gamma$. How do you incorporate this Neumann boundary condition into Galerkin's method? Show all details, including the relevant matrix equation(s).