Math 187 Test III

Dr. Holmes

The test will begin at 11:40 am and end at 12:35 pm. You may use a plain scientific calculator without graphing or symbolic computation capabilities. More competent calculators, cell phones, or PDA’s may not be used; all cell phones must be turned off and out of sight. Books, notes, and neighbors to remain firmly closed.
1. Apply the sieve of Eratosthenes to find the prime numbers less than 20. Your work should make it clear that you know how the sieve works; English explanation could help.
2. Write out the multiplication table for the arithmetic of congruence classes modulo 7. Make a table of the multiplicative inverses of the congruence classes. You may write the congruence classes just as the numerals from 0 to 6.
3. “Division” of congruence classes and the Chinese Remainder Theorem.

(a) Solve the equation
\[ 45x \equiv 32 \pmod{101} \]

(b) Find the smallest solution to the system of equations
\[ x \equiv 31 \pmod{121} \]
\[ x \equiv 25 \pmod{256} \]
4. Do two of the three mathematical induction proofs. You get some credit for correctly setting up the problem even if you do not see how to solve it.

(a) Prove by mathematical induction that

\[ \sum_{i=1}^{n} (2i - 1) = n^2 \]

(the sum of the first \( n \) odd numbers is \( n^2 \)). Make sure that your basis step, induction step and induction hypothesis are clearly identified and that you indicate where the induction hypothesis is used.
(b) Prove by mathematical induction that $2^n > n^2$ for each $n \geq 5$. 
(c) Prove by mathematical induction that $8^n - 3^n$ is divisible by 5 for each $n$. 
5. Arithmetic and geometric sequences

(a) Determine the 37th term of the arithmetic sequence

6, 13, 20, ...

Determine the sum of the first 37 terms of this sequence without adding them all up (show work).

(b) Determine the 5th term of the geometric sequence

75, 45, 27, ...

What is the sum of the first 10 terms (show how you do this without adding them all up)? What is the sum of all terms of this sequence?
6. Recursively defined sequences

(a) Determine the next five terms of the sequence whose first two terms are 1 and 2 and which satisfies the recurrence relation

\[ a_n = a_{n-1}^2 - a_{n-2}. \]

(b) Find a closed form formula for \( b_n \), where \( b_0 = 1 \), \( b_1 = 1 \), and

\[ b_{n+2} = 3b_{n+1} - 2b_n. \]
7. Of 389 schoolchildren, all of whom like at least one of the standard flavors of ice cream, 295 like vanilla, 342 like chocolate, and 249 like strawberry ice cream. 252 children like vanilla and chocolate; 228 like vanilla and strawberry; 227 like chocolate and strawberry.

Use the principle of inclusion-exclusion to determine how many children like all three flavors. Show the complete inclusion-exclusion calculation so that I can tell that you understand the principle.