1. Some linear orders are listed. For each one, state that it is a well-ordering or that it is not. If it is not, explain precisely why it is not (this means give an example of something). If it is, give its order type (an ordinal number).

(a) $\emptyset$

(b) the standard order on the integers restricted to $\{x \in \mathbb{Z} \mid -2 \leq x \leq 2\}$

(c) the standard order on the integers restricted to $\{x \in \mathbb{Z} \mid x \leq 0\}$

(d) the standard order on the rationals restricted to $\left\{ \frac{n}{n+1} \mid n \in \mathbb{N} \right\} \cup \{1\}$

(e) the standard order on the rationals restricted to $\left\{ \frac{n+1}{n} \mid n \in \mathbb{N} \right\} \cup \{1\}$

(f) the standard order on the reals restricted to the interval $[0, 1]$

2. Prove that for any natural number $n$, any two linear orders with a field of size $n$ are isomorphic, and all such linear orders are well-orderings. (How do we prove anything about natural numbers?)

3. Using the definitions in the notes, prove that if $R$ and $S$ are well-orderings, so is $R \oplus S$. You need to prove that it is a linear order (which will probably require some reasoning by cases) and prove that it has the additional defining property of a well-ordering.

Now that you are filled with self-confidence, do the same for $R \otimes S$. 

4. Define sets of real numbers such that the restriction of the standard order on the real numbers to that set has each of the following order types:

(a) $\omega + 1$
(b) $\omega \cdot 3$
(c) $3 \cdot \omega$
(d) $\omega \cdot \omega$
(e) $\omega \cdot \omega \cdot \omega$ (OK I suppose this is nasty, but see if you can do it)

5. Prove your choice of the two following annoying propositions (these are annoying in the sense that they are straightforward (even “obvious”) but there is a good deal to write down).

(a) Isomorphism is an equivalence relation on relations.
(b) A relation isomorphic to a well-ordering is a well-ordering.