# Math 147 Final Exam

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This exam is open books, open notes, any calculator, closed neighbor. It begins officially at 10:30 am and ends officially at 12:30 pm; I am willing to allow the exam to continue until 12:45 pm, if no student objects.

Be sure to show all work. When you use your calculator, show your set-up and sketch any graphs you use.

1. Find the domains of the functions.

   (a)
   $$f(x) = \frac{x-2}{x^2-5x+6}$$

   (b)
   $$g(x) = \sqrt{4-x^2}$$

   (c)
   $$h(x) = \log_7(2-x)$$

2. Some graphs are given below. For each graph, determine whether it is the graph of a function or not. If it is not the graph of a function, explain why not. If it is the graph of a function, determine whether the function has an inverse or not. If it does not have an inverse function, explain why not. If it does have an inverse function, sketch the graph of the inverse function on the same pair of axes.

3. Extreme value word problems

   (a) A farmer wishes to enclose a field next to a highway. Zoning reglations require that the field be rectangular. Highway regulations require that he use a special fence costing 2 dollars per yard on the highway side of the field; he can use a cheaper kind of fence costing 50 cents per yard on the other three sides. He has $10,000 to spend on fence. What are the dimensions of the largest field he can enclose under these conditions?

   You need to show all work and give an exact answer (graphing calculator use not appropriate except to check).

(b) (graphing calculator) A three by five inch card is to have squares cut out of the corners so that it can be folded into a tiny box. Determine the volume of the box as a function of the length of the sides of the cutout squares. Use your graphing calculator to estimate the dimensions (length, width, and height) of the largest possible box which can be constructed in this way. Estimate to within a tenth of an inch.

4. Graphing of polynomials and rational functions. In both parts of this problem, the graphs are qualitatively correct but not necessarily to scale. In both parts, you can leave polynomials in factored form – there is no need to expand.

   (a) Write a formula for the polynomial with the given graph. It has no $x$-intercepts other than those shown, and the coefficient of the highest power is 1.

(b) Write a formula for the rational function whose graph is shown. Hint: be sure to take advantage of the vertical asymptotes and $x$-intercepts shown to determine factors of the numerator and denominator of the expression defining the function (not necessarily in that order). You can also get information about the function from its $y$-intercept and horizontal asymptote.

5. Applications of synthetic division

(a) Let

$$f(x) = 3x^4 - x^2 + 5x - 1.$$

Determine $f(2)$ using synthetic division. Show your work.

(b) I give you for free the information that 2 is a root of the polynomial

$$g(x) = x^4 - 3x^3 + 3x^2 - 3x + 2.$$

Factor this polynomial completely and determine all its real and complex zeroes. I reiterate that a complete answer has two parts: you need to give the polynomial in factored form and give a list of its real and complex zeroes. I suggest using synthetic division appropriately.

6. Perform the indicated calculations with complex numbers.

(a)
$$(3+i)(1-2i)$$
Write your answer in the form $a+bi$, where $a$ and $b$ are real numbers.

(b)
$$\frac{1+2i}{3-2i}$$
Write your answer in the form $a+bi$, where $a$ and $b$ are real numbers.

(c) Convert the complex number $1+i$ to trigonometric form and compute $(1+i)^5$ using de Moivre's theorem. You may leave your answer in trigonometric form.

7. Compound interest, exponential growth and decay. You may choose one of the two problems.

   (a) If you invest \$10,000 at 8% compounded quarterly, how much money will you have after 10 years? How long will it take for your investment to triple?

   (b) A population of bacteria is growing exponentially. At the beginning of the experiment, there are 10,000 bacteria. After six hours, there are 450,000 bacteria. Write a formula for the number of bacteria after $t$ hours. How long will it take before there are 1,000,000,000 bacteria in the culture?

8. Logarithms.

   (a) Rewrite

   $$\log\left(\frac{x^2}{y\sqrt{z}}\right)$$

   in terms of simpler logarithms. Your answer should not contain any logarithm of a product, quotient, power, or root.

   (b) Rewrite

   $$\ln(x) - \frac{1}{3}\ln(y) + 5\ln(z)$$

   as a single logarithm.

9. A right triangle is shown. Determine the values of all six trigonometric functions at the labelled angle $\theta$.

   Then determine the values of all six trigonometric functions at the angle $2\theta$.

10. Solve the triangles shown, using the Law of Sines or the Law of Cosines as appropriate. The triangles are definitely not to scale.

11. Verify the trigonometric identities. Show all work.

(a)

$$1 + \sin(x) = \frac{\cos^2(x)}{1 - \sin(x)}$$

(b)

$$(\sin(x) + \cos(x))^2 = 1 + \sin(2x)$$

12. Determine the inverse of the matrix, then use a calculation with the inverse matrix to solve the system of equations. You are required to find the matrix inverse by hand, showing all row operations. You are required to set up the matrix calculation that solves the equation on your paper, but you may use your calculator to carry it out (it's not especially hard to do it by hand, though).

    The matrix:

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & -1 & 1 \\ -2 & 2 & -1 \end{bmatrix}$$

    The system of equations:

$$x + 2y + 2z = 3$$
$$2x - y + z = 6$$
$$-2x + 2y - z = -3$$