Math 170 Test I

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This test will begin at 9:35 and end at 11:35. You may use any calculator (in fact, you need a calculator!). You may not use books, notes or neighbors. You must show all work and certainly any specific calculations I request for full credit (I am aware that powerful calculators will answer some of these questions with a minimum of thought on your part!)

1. The point $P = (2, 3)$ lies on the graph of the function $y = \sqrt{2x + 5}$. If Q is the point $(x, \sqrt{2x + 5})$, determine the slope of the secant line PQ for each of the following values of x. Do all calculations to as many decimal places as your calculator will display.

   (a) 2.01

   (b) 1.99

   Guess at the slope of the tangent line to the graph at $(2, 3)$. How could you improve your guess?

2. Give the appropriate answer to each limit question, either a definite value, or ∞, or $-\infty$, or "does not exist". In each case, you must show appropriate calculation and/or explanation to support your answer (the picture on your graphing calculator is not an explanation).

(a)

$$\lim_{x \to -3} \frac{x^2 - 9}{x - 3}$$

(b)

$$\lim_{x \to 2} \frac{x - 2}{x^2 - 4}$$

(c)

$$\lim_{x \to 1+} \frac{1 + x}{1 - x}$$

3. Verify that $f(x) = \sqrt{x+5}$ is continuous at 4 using the definition of continuity and the limit laws. That is, express the statement that this function is continuous at 4 as a statement about limits, then prove that the statement is true using the limit laws (no ϵ or δ should be involved).

4. Sketch the graph of a function which satisfies

$$\lim_{x \to 0-} f(x) = -\infty,$$

$$\lim_{x \to 0+} f(x) = 0,$$

$$\lim_{x \to 1-} f(x) = 1,$$

$$\lim_{x \to 1+} f(x) = 1,$$

$$f(1) = 2,$$

is continuous everywhere except at 0 and at 1, and is continuous from the right at 0.

Which, if any, of the discontinuities of this function are removable?

5. Determine all vertical and horizontal asymptotes of the function

$$f(x) = \frac{x^3 + 1}{x^2(3x - 6)}.$$

Give all the appropriate statements involving infinite limits or limits at infinity which describe these asymptotes (including one-sided infinite limits where appropriate).

6. The height of a falling object above the ground is given by

$$h = 500 - 16t^2.$$

Time is measured in seconds.

Set up and evaluate a limit (showing all work) representing the velocity of this object when $t = 3$.

Use the result of your calculation to write an equation for the tangent line to the graph of $y = 500 - 16x^2$ at the point with x-coordinate 3.

7. State the definition of the derivative $f'(x)$ of a function f (you may use either of the two forms I have used).

Determine the derivative $f'(x)$ of the function $f(x) = \frac{1}{x^2}$, using the definition of the derivative as a limit, and use the result to write an equation for the tangent line to this curve at $(\frac{1}{2}, 4)$.

Suppose that the tangent line to the graph of f at $(2, 5)$ passes through the origin. What is $f'(2)$?

8. Prove that

$$\lim_{x \to 2} 3x + 1 = 7,$$

using the official definition of limit (with ϵ and δ).