Math 333 Final Exam

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You have from 8 am to 10:15 am to take the exam (if everyone present agrees to extend the time to 10:15 am from 10 am). You are permitted to use your book and notes, and any calculator. You may not use palmtops or cell phones as calculators, for security reasons. Please be sure that you show all work that is requested. Any calculations that you do on your calculator should be set up on your paper.

The values of the numbered questions are equal.

1. Solve the separable differential equation. Be sure to give the singular solution or solutions, or state that there isn't one (and why).

$$y' = xy - x$$

2. Solve the linear differential equation.

$$tx' = 4x + t^4$$

3. Solve the initial value problem.

$$y'' + 25y = 0; y(0) = 0; y'(0) = 1$$

4. Solve the initial value problem using the method of undetermined coefficients. Show all work.

$$y'' - 5y' + 6y = \sin(t); y(0) = 0; y'(0) = 0$$

5. Solve the initial value problem using the method of Laplace transforms. Show all work.

$$y'' - 5y' + 6y = 0; y(0) = 1; y'(0) = -1$$

6. Draw arrows in the direction field of this system of equations at $(1,1), (1,-1), (-1,1), (-1,-1)$. Show supporting computations for these arrows.

   Use the roots of the characteristic equation to classify the equilibrium point at (0,0) (as nodal source, nodal sink, spiral source, spiral sink, center or saddle point – you must explain what facts about the roots justify your classification). Then give a qualitatively accurate sketch of the whole direction field and a typical solution curve.

$$x' = -x - 2y$$
$$y' = 2x - y$$

7. Give the general solution to the homogeneous system of linear equations. Show all work, including hand calculation of eigenvectors and eigenvalues.

$$x' = 2x - y$$
$$y' = x + 4y$$

8. Solve the inhomogeneous linear system. You may use the method of variation of parameters or the method of undetermined coefficients. Your final answer should be the general solution, in the form of an expression for $x(t)$ and an expression for $y(t)$ (i.e., not in vector form).

$$x' = 2x + y + e^{2t}$$
$$y' = x + 2y - e^{2t}$$

A basic set of solutions to the homogeneous equation (given in vector form) consists of

$$e^{3t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

and

$$e^{t}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$