Math 170 Final Exam, summer 2002

Dr. Holmes

July 31, 2002

Time available to do the exam is from 9:05 am to 12:05 pm. The exam is not written as a three-hour exam; the idea is that time should not be the limiting factor. If you arrive at 9:35, you still have plenty of time!

You may use your book and any notes you wish to use. You may not use a calculator with graphing or symbolic computation capabilities.

There is a serial number on your paper, which is the code I will use to post your final exam and course grades on my web page.

1. Estimate

$$\ln(3) = \int_1^3 \frac{1}{x}\, dx$$

by setting up a left sum with four subdivisions to estimate the definite integral.

Compute your estimate, writing down all details of your sum and the answer to four decimal places.

Sketch the region whose area is given by the integral and the "boxes" whose area adds up to the Riemann sum. Is your estimate an overestimate or an underestimate?

2. Find the antiderivatives. Show all work (especially if you use the method of substitution).

(a)

$$\int \frac{x^2 + x + 1}{x}\, dx$$

(b)

$$\int \sqrt[3]{x^2} - \sqrt{x^3}\, dx$$

(c)

$$\int \sec(x)\tan(x)\, dx$$

(d)

$$\int 3t\cos(t^2)\, dt$$

3. Find the definite integrals. Show all work (especially if you use the method of substitution).

(a)

$$\int_1^2 (x^2 - 4)\, dx$$

(b)

$$\int_0^{\frac{\pi}{4}} \sin(4t)\, dt$$

(c)

$$\int_1^e \frac{\cos(\ln(x))}{x}\, dx$$

(d)

$$\int_0^1 \frac{3x^2}{1 + x^6}\, dx$$

Hint:

$$u = x^3$$

4. The velocity of a particle at time t is given by the formula

$$v(t) = 9 - t^2.$$

(a) Determine how far the particle is at time $t = 5$ from its position at time $t = 0$.

(b) Determine the total distance travelled by the particle between time $t = 0$ and time $t = 5$.

5. Compute the derivative of the function $f(x) = x^2 - x$ using the definition of the derivative as a limit. Show all work. (Do not use L'Hôpital's Rule to evaluate the limit, but don't use epsilons and deltas either!)

6. Determine the derivatives, showing all work. One part has additional instructions. There is no need to simplify once all derivatives are computed.

(a)

$$f(t) = \sqrt{t} - \frac{1}{\sqrt{t}}$$

(b)

$$g(x) = \frac{e^x}{x + e^x}$$

(c)

$$j(x) = e^x \sin(x)$$

(d)

$$k(x) = \ln(x + \ln(x))$$

(e)

$$m(x) = \sqrt[3]{\frac{x^2 + 1}{x^2 - 1}}$$

(use logarithmic differentiation)

7. Find the tangent line to the curve

$$x^5 + y^5 = 33$$

at $(1, 2)$ using implicit differentiation.

8. Determine intercepts, asymptotes, intervals of increase and decrease, local maxima and minima, intervals of concavity up and down and points of inflection for

$$f(x) = \frac{x}{x^2 + 1}.$$

Write appropriate statements about limits at infinity or infinite limits for each asymptote.

Sketch the graph of the function, labelling significant points with x and y coordinates.

I give you the first and second derivatives of f:

$$f'(x) = \frac{1 - x^2}{(x^2 + 1)^2}$$

$$f''(x) = \frac{2x^3 - 6x}{(x^2 + 1)^3}$$

9. Word problems: do one of the three problems. If you do more, your best work will count.

   (a) A building is 30 feet from a searchlight at ground level. A woman who is 5 feet tall walks from the building toward the searchlight at 4 feet per second. How fast is her shadow moving up the wall when she is 10 feet from the searchlight?

(b) A box with an open top is to be made with cardboard sides and a square metal base. The cardboard costs 3 cents per square inch. The metal material for the bottom costs 10 cents per square inch. The total volume of the box is to be 1000 cubic inches. What should the dimensions of the box be to minimize cost?

(c) A retailer buys widgets at ten cents each. He knows that if he buys one thousand widgets, he can sell them at fifteen cents each, and that for each additional thousand widgets that he sells, the price he will be able to get will go down by one cent. How many widgets should he order for maximum profit?