

# Final Review Sheet, Math 301

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## 1 General Remarks

**NEW AND IMPORTANT:** I forgot to mention that as a rule my exams are **OPEN BOOK** and **OPEN NOTES**, and this exam is no exception. Please note this, and send me an e-mail to tell me that you have been informed if you haven't already done so!

The exam will have no more than 12 questions; fewer if I can manage it.

Tests I-IV already given should provide good models for questions; I have no intention of being "tricky". But there will be proofs (as the tests already given should indicate). A likely format for proofs (as for word problems) is that you may be given a selection of problems from which you may choose one (or do more than one and get the best one graded).

The questions will be taken from the entire course, but the intention is not to ask about *every* possible topic. As a result, there may be topics mentioned in the outline below which do not appear on the final at all!

Nothing I say here in any way limits what I may ask on the hardest question on the exam (the A/B separator). I always make this exception but I don't necessarily take advantage of it!

You might want to check at my door for copies of the solution set for Test IV, which I keep forgetting to run off!

## 2 Review of Sections

**sections 1-4:** Be familiar with the equivalent forms in which systems of linear equations can be presented: be able to convert freely between systems of linear equations, vector equations, and matrix equations.

Be able to use reduced row echelon form to solve such equations.

On most of the exam, you will be able to use your calculator to find reduced row echelon forms, but there will be at least one question from Chapter 1 about the details of this process: you need to be able to do it by hand, explicitly describing row operations and identifying such details as pivot columns, basic variables and free variables.

**section 1.5:** Be aware of the geometric structure of solution sets and the relationship between existence of free variables and rightmost pivot columns on the one hand and existence and uniqueness of solutions on the other. Be able to write solution sets in parametric vector form.

**section 1.6:** I might ask network flow or chemical equation balancing word problems. There is likely to be a choice between various kinds of word problems.

**section 1.7:** Know the definition of linear independence and be able to determine whether sets of vectors are linearly independent. This topic may be entirely subsumed into questions about the concept of basis.

**section 1.8-9:** Know what a linear transformation is. Know what is involved in proving that a function is a linear transformation. Know how to determine the matrix of a linear transformation described geometrically. Know how to determine the domain and range of a matrix transformation, and how to determine whether it is one-to-one and whether it is onto.

**section 1.10:** I could ask an electric circuit question (as one of a number of word problems from which you get to choose).

**section 2.1-3:** Know how to multiply matrices. You should be able to understand the relationship between matrix multiplication and composition of linear transformations represented by matrices. You should be familiar with the details of different definitions of matrix multiplication.

Be able to do algebra of matrices, involving both products and inverses (and also the rules for transposes). Remember that multiplication of matrices is not commutative!

Know the definition of the matrix inverse and how to compute matrix inverses using row reduction of a suitable matrix.

I doubt that I will ask anything directly about elementary matrices, though this concept might be useful in some proof.

You are responsible for the properties of matrices summarized in the version of the Invertible Matrix Theorem found in section 2.3 and also for all later additions to this Theorem!

**section 2.4:** If I ask anything about partitioned matrices, it will be strictly for the sake of the exercise in matrix algebra.

**section 2.5:** I will not ask about LU factorizations.

**section 2.8-9:** see chapter 4.

**section 3.1:** Be able to compute determinants by cofactor expansion. Be able to compute two by two and three by three determinants using their appropriate special methods.

**section 3.2:** Be familiar with the effects of row operations on matrices on the values of their determinants. be aware that a matrix is invertible iff its determinant is nonzero. Be aware that the determinant of the product of two matrices is the product of their determinants.

**section 3.3:** Cramer's rule, which I did not cover on the hour exam, well might be on the final. The adjoint method of computing inverses might be covered, though it is rather laborious. Areas of parallelograms and volumes of parallelepipeds are fair game.

**section 4.1:** Know the definition of a vector space. Know what goes into proving that a set with given addition and multiplication operations is or is not a vector space.

**section 4.2:** Know about null spaces, column spaces, and general kernels and ranges of linear transformations. Be able to present a null space in parameterized form. There's also stuff in 2.8-9 about this.

**section 4.3:** Know the definition of a basis and the major theorems about bases (here and later – I'm thinking of theorems about dimension, for example).

**section 4.4:** Know how to find coordinates for a given vector with respect to a given basis. Be able to write the change-of-coordinates matrices for a basis (either to or from standard coordinates).

**section 4.5-6:** Know what dimension is. Know what rank is. Know the relationship expected between dimensions of null spaces and column spaces (and between null spaces of transposes and column spaces of transposes: I will accept confusion between the row space and the column space of the transpose for this exam).

**section 4.7:** Be able to change between two different bases on a vector space: this is really the same ideas as in section 4.4, plus keeping your wits about you. I also note that I may stick to 4.4 questions unless I come up with a question I really like. Be able to write matrices representing change of basis if I do ask a question from here.

**section 4.8-9:** There will be no mandatory coverage of these sections – it is possible that a 4.9 question might appear as one of the word problems from which you can choose.

**chapter 5:** The coverage will be exactly as on Test IV, except that section 5.8 will not be covered. I covered more chapter 5 material than chapter 6 material on test 4; the reverse is likely to be true on the final. I am very likely to ask a question like problem 3, test 4 again, so be sure to be ready for it this time!

**chapter 6:** The concepts of orthogonal bases, orthogonal projections onto vectors or subspaces, and the Gram-Schmidt process will be covered, as in Test 4. There is new material, of course: you should expect a 6.5 question, which might take the form of a curve-fitting question as in section 6.6, and you should expect a question from 6.7 or 6.8. I really like the Fourier approximation topic from 6.8: if I give a problem like that, it will be exactly like the example I did in class (give an estimate of such-and-such order of such-and-such a function).