Math 187 Test III

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The test begins at 7:50 am and ends at 9:35 am. You are permitted to use a calculator; please set up any calculations you do with the calculator on your paper so I can tell how you got your answers. In general, show your work.

You may omit one question; if you do all questions, the lowest will be dropped.

   Carry out the addition of these two numbers in base 7, convert back to base 10, and check your answer.

   Carry out the multiplication of these two numbers in base 7, convert back to base 10, and check your answer.
2. Compute the greatest common divisor of 406 and 518, using the Euclidean algorithm.

Using information from the calculation of the gcd, determine integers $m$ and $n$ such that $406m + 518n = 56$, if this is possible. If it is not possible, explain why not.
3. Repeating decimals (all in base 10!)

   (a) Compute the decimal representation of \( \frac{3}{37} \) (show your long division).

   Give fractions bounding \( \frac{3}{37} \) above and below as closely as possible with denominators 100, 10,000 and 1,000,000.

(b) Express the repeating decimal 1.232323... as a fraction (improper fractions are fine). You must show calculations supporting your answer.
4. There are 25 students in a high school class. French, German, and Russian are the foreign languages offered. 5 students take no foreign language. There are nine students who take French, nine who take German, and 12 who take Russian. There are three who take both French and German, five who take both German and Russian, and three who take both French and Russian. How many students take all three languages?

A Venn diagram with as much information as you can find would be useful. You will not receive full credit without an explanation of how you got your answer.
5. A state uses license plates that consist of four letters followed by three digits. Answer the following questions (show your calculations!)

(a) How many possible license plates are there?

(b) How many possible license plates contain at least one 7 (Think about ones that don’t contain any 7’s).

(c) How many possible license plates are there if no letter or digit may be used more than once?

(d) How many possible license plates are there if no letter or digit may be followed immediately by the same letter or digit?

(e) How many possible license plates contain exactly one letter A, if no letter or digit may be used more than once?
6. Sophisticated computations. You may do one of the following; if you do both, the best will count.

(a) Determine the repeating “decimal” expansion for $\frac{2}{7}$ in base 9.

(b) Compute the square root of 3 to three decimal places using the “long division” algorithm introduced in class.
7. Give a proof that if $c|a$ and $c|b$, then $c|2a + 3b$. Your proof may assume knowledge of algebraic properties of the integers, but you may not assume that you know any properties of divisibility except the definition (i.e., don’t assume that the sum of two numbers divisible by $c$ is divisible by $c$ – show that this is true by calculation).
8. The multiplicative inverse property of the rational numbers asserts that each rational number \( x \neq 0 \) has a multiplicative inverse \( x^{-1} \) such that \( (x)(x^{-1}) = 1 \).

If \( x \) is represented as \([ (a, b) ]\) in our construction, how is \( x^{-1} \) represented? (If you interpret this in terms of fractions, this is easy).

Why does your definition of \( x^{-1} \) not work if \( x \) is the rational number 0?

Verify that \((x)(x^{-1}) = 1\) using your representation of \( x^{-1} \) and the definitions of multiplication and equality in our construction of the rationals (given below).

The information that you need about the rational number construction follows:
Recall that a rational is an equivalence class \([ (a, b) ]\) of ordered pairs \((a, b)\), where \( a \) is an integer and \( b \) is a nonzero integer (we actually required \( b > 0 \) as well, but allowing negative \( b \) makes this problem easier).

\([ (a, b) ] = [ (c, d) ] \) when \( ad = bc \) (this defines the equivalence relation).

We define the product \([ (a, b)][(c, d)]\) of two rational numbers as \([ (ac, bd) ]\).

The rational number 1 is \([ (1, 1) ]\).