Review for Test III, Math 301

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Permitted resources will be the same as on previous exams. You will be permitted to use any calculator. What you are actually allowed to do with the calculator is compute reduced row echelon forms of matrices, compute matrix products and inverses, and compute determinants. Pay careful attention to instructions where you may be told to carry out certain calculations by hand. You will not be allowed the use of books and notes. Certain kinds of information may be given to you with your test (for example, the axioms of a vector space, if needed, will be provided).

There will be no more than 8 questions on the exam (though they may have parts), and there are 13 sections covered. Remember that much of chapter 4 turned out to be review of stuff we did in chapter 2, so there will not be a question per section in chapter 4: there will be questions mostly on new ideas. This does not mean that you do not need to have the skills from chapter 2: in particular, you certainly need to be able to find bases of column spaces and null spaces.

For theory, the true-false questions in all sections might be good for study. The blocks of questions I mention are quite large – the point being that I'm indicating kinds of questions that might be asked, rather than how many problems you should do while reviewing.

section 3.2: The emphasis here is on properties of determinants. You should be able to compute a determinant by using row operations, and you should understand the effect of any row operation on the value of a determinant. You should know that the determinant of the product of two matrices is the product of the determinants of the two matrices, and you should know the linearity property (if any row or column of a matrix is multiplied by a constant, the effect on the determinant is multiplication by the same constant). You should know that a matrix is invertible iff it has a nonzero determinant.

Exercises like 15-18 are good for properties. Think about 31, 32, 41,43 (though 43 as it stands involves too much calculation). More computational, problems like 5-14. Supplemental exercises: 2-4, 7,8.

section 3.3: You should know Cramer's rule and you should know the formula for matrix inverses using the "adjugate". Questions involving either of these might just involve setup, depending on how complex they are, but you should be able to compute answers (with help of a calculator).

You should know the relationship between determinants and areas of parallelograms (and triangles) and volumes of parallelipipeds. Be aware of the effect of any linear transformation on areas of arbitrary figures.

Computational examples, 1-6, 11-16, 19-24 (areas and volumes). Supplemental: 11.

remark about chapter 4: Much of the material in chapter 4 is review from chapter 2. Certain sections from chapter 4 might not end up being covered on the test at all, for this reason.

section 4.1: If you need the list of axioms of a vector space (which you well might) it will be supplied.

I might ask you to prove something like $0\mathbf{x} = \mathbf{0}$ from the axioms (or this might be an extra credit opportunity). Alternatively, some kind of vector space proof might be supplied and you might be asked to supply justifications of steps.

Be able to verify that a given set with given operations is a vector space (or that it is not a vector space). Most likely, such a set will be a subset of a larger vector space, so there will be only three of four things to verify.

problems 1-4 are good for review, as are 5-14, 15-18. 25-30 for proofs.

section 4.2: Facts about null spaces and column spaces of matrices remain fair game.

Computational questions can look like 1-6, 7-14, 17-24 (this is not exhaustive).

You need to know what the kernel of a linear transformation is (of course you know what the range is). For this, look again at problems 31-32. 34-36 are also worth looking at.

section 4.3: Be familiar with the definition of a basis. Know how to find bases of null spaces and column spaces of a matrix (we have already done this).

Know the spanning set theorem (along with later theorems).

Any of the computational questions at the beginning of the homework would be fair game (1-20). 31-32 are good to look at.

section 4.4: Know the unique representation theorem. Understand how to get coordinate mappings from a basis to the standard coordinates and from the standard coordinates to coordinates in the terms of the basis.

certainly 1-12, also 13, 14, 27-34.

section 4.5: Know the theorems (9,10,11,12). Know the relationship between the dimensions of the null space and column space of a matrix.

Know the difference between a finite- and an infinite-dimensional space.

Problems 11-18 – find bases as well as stating dimensions. problems 21-24. Problem 27 or 28.

section 4.6: This section is almost entirely review: but you do need to know facts about row spaces, which are first introduced here. Know how to find a basis for a row space, and the relation between dimensions of row space and null space.

You do need to know what rank is, of course.

section 4.7: Be able to do this stuff! Problems 1-10, 13, 14.

section 4.8: Be able to use a linear difference equation to compute terms given starting values. Be able to find r such that the sequence r^k is a solution to a linear difference equation. Be able to say something about whether given solutions to a linear difference equation are linearly dependent or independent (or whether a set of solutions is a basis for the space of all solutions).

Problems 3-16, 25-28 (these are inhomogeneous).

section 4.9: Be able to set up and carry out experiments with Markov chains on your calculator. Be able to solve for the fixed point (steady state vector) of a stochastic matrix.

Problems 1-8, 15, 16.

section 5.1: Be able to determine whether a given vector is an eigenvector of a given matrix, and if it is, what eigenvalue it represents.

Be able to determine the eigenspace of a given eigenvalue of a given matrix: be able to find a basis of eigenvectors for that eigenvalue.

Be able to prove that the eigenspace of an $n \times n$ matrix is a subspace of R^n.

Problems 1-20, 31, 32, 37-40.

section 5.2: Be able to determine the characteristic polynomial of a 2×2 or 3×3 matrix (calculator use allowed as long as you set up the determinant to be evaluated on your paper). In the 2×2 case, be able to solve for the eigenvalues (they will be real!) and find eigenvectors representing them.

Problems 1-14. In problems 1-8, do the additional work of finding eigenvectors.