Math 170 Makeup for Test III

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You have the entire period to work on this makeup. Nothing you do or fail to do can lower your grade on the exam. You may leave when finished. The usual rules about calculators and cell phones apply.

You can make up either problem 1 (linearization and approximation) or problem 8 (L'Hôpital's Rule) Your best applicable problem will replace the problem on your test if it helps you.

You can make up one curve sketching problem: there will be one like problem 5 and one like problem 6, and your best performance will help you. If you do both of these and do well on both, you may earn some extra credit.

You can make up the word problem: two word problems are supplied, and your best performance will replace problem 7 if this helps you. If you do both word problems and do well on both you may earn some extra credit.

Your adjusted grade for Test III will be posted on my web page using the magic number on this makeup paper (whether it changes or not).

1. (problem 1) Find the linear function which best approximates $y = \sqrt{x}$ near $x = 25$ (hint: its graph is the tangent line to $y = \sqrt{x}$ at (25,5), but be sure to write it as a function), and use this function to approximate $\sqrt{24.8}$.

2. (problem 8) In each part, apply L'Hôpital's Rule if you can (and as many times as you can), indicating why the rule is applicable or why it is not (arrows indicating the limits of numerators and denominators work fine for this); evaluate the limit whether you can apply the rule or not.

   (a)
   $$\lim_{x\to 1} \frac{x^3 - x}{x^3 - 1}$$

   (b)
   $$\lim_{x\to 0} \frac{\cos(2x)}{\cos(3x)}$$

   (c)
   $$\lim_{x\to \infty} \frac{x^2}{e^{2x}}$$

3. (problem 5)

The function

$$f(x) = \frac{x^2 - 4}{(x-4)^2}$$

has the following first and second derivatives:

$$f'(x) = \frac{-8(x-1)}{(x-4)^3}$$

$$f''(x) = \frac{8(2x+1)}{(x-4)^4}$$

I tell you for free that this graph has a horizontal asymptote at $y = 1$, a vertical asymptote at $x = 4$, and x-intercepts at $x = 2$ and $x = -2$.

Using a sign chart for f, determine where the graph of f is above the x-axis and where it is below (this will tell you how it approaches the asymptotes)

Using a sign chart for f'', determine intervals on which f is increasing and intervals on which f is decreasing. Identify any local maxima or minima of f.

Using a sign chart for f'', determine intervals on which the graph of f is concave up and intervals on which it is concave down. Identify any points of inflection on the graph.

Use this information to sketch the graph of f on the following blank page (feel free to tear it out if this makes it easier for you to work; just make sure you write your name on it if you do this).

4. (problem 6)

We are given the following information about a function f. Notice that information alternates with questions for you to answer.

$$\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 0$$

$$\lim_{x\to 3-} f(x) = -\infty; \lim_{x\to 3+} f(x) = \infty$$

Describe the horizontal and vertical asymptotes of the the graph of f.

$f(x) > 0$ for $x < 1$; $f(1) = 0$; $f(x) < 0$ for $1 < x < 3$; $f(x) > 0$ for $x > 3$

$f'(x) > 0$ for $x < 0$; $f'(0) = 0$; $f(0) = 2$; $f'(x) < 0$ for $0 < x < 3$; $f'(x) < 0$ for $x > 3$

Describe intervals on which the function increases and intervals on which it decreases and identify any local maxima or minima.

$f''(x) > 0$ for $x < -2$ and for $x > 3$; $f''(x) < 0$ for $-2 < x < 3$; $f(-2) = 1$

Describe intervals in which the graph of the function is concave up and intervals on which the graph is concave down and identify any points of inflection.

Use the information given to sketch the graph of f on the following blank page (feel free to tear it out if this makes it easier for you to work; just make sure you write your name on it if you do this).

5. (problem 7a)

   A packing box is made with top and bottom plain cardboard that costs one cent per square inch and four sides made of printed cardboard that costs two cents per square inch. The volume of the box is to be 4000 cubic inches. What should the dimensions of the box be to minimize the cost of materials?

6. (problem 7b)

   A girl sells seashell necklaces at the beach. If she charges twenty dollars per necklace, she will sell thirty necklaces in a day. For each dollar that she lowers the price, she will sell four more necklaces. It costs her five dollars to make each necklace. How many necklaces should she sell each day at what price for maximum profit?

   Remember that profit is revenue minus cost, that revenue is number of items sold times unit price and cost is number of items sold times unit cost. I would use the variable x equal to the number of dollars by which she lowers the price.