Math 170 Test I

Dr. Holmes

The exam will begin at 10:40 am and end at 11:35 am. You will need a plain scientific calculator without graphing or symbolic computation capabilities. You may not use a cell phone, PDA, or more capable calculator for this purpose. Books, notes and neighbors to remain firmly closed.

Good luck!

1. Estimation of velocity

   Suppose that the position of an object at time t seconds after a fixed starting time is $t^2 - t$ feet along a straight track from a reference point.

   Determine the average velocity of the object during the interval of time $[2, 2.1]$. Determine the average velocity of the object during the interval of time $[1.9, 2]$.

   Compute some average velocities over "better" intervals (tell me which ones) and estimate the instantaneous velocity of the object at $t = 2$. Use at least two "better" intervals and don't neglect the time before $t = 2$ or the time after it.

2. Limits

Estimate the limit of $\lim_{t\to 0} \frac{\tan(t)}{t}$ by computing relevant values of this function with your calculator. Be sure to have your calculator in radian mode!

Explain why we cannot evaluate this limit using the limit law for the quotient.

Compute the following limits. You do not need to use epsilons and deltas, nor do you need to do complete step-by-step application of the limit laws, but you do need to show work (including required comments).

(a)
$$\lim_{x\to 3} \frac{x-2}{x^2-4}$$

(b)
$$\lim_{x\to 2} \frac{1}{\sqrt{x+2}+2}$$

(c)
$$\lim_{x\to \infty} \frac{2x+3x^2}{5x-2x^2-1}$$

3. Express the assertion that $f(x) = \frac{x+1}{x-1}$ is continuous at 2 using the definition of continuity: this will be an equation involving a limit on one side and a value of the function f on the other.

   Then verify the correctness of this equation using step-by-step application of the limit laws (this does not involve any epsilons or deltas!)

4. State the limits at infinity or infinite limits (including one-sided and two-sided infinite limits) of the function f whose graph is shown. Hint: at least one infinite limit or limit at infinity is associated with each asymptote pictured.

5. The graph of a function f is shown. Identify the x values at which f is discontinuous. State what kind of discontinuity there is at each of these values. Identify any of these values at which the function is continuous from the left or right, and list the intervals on which the function is continuous.

6. Determine the tangent line to the graph of $f(x) = x^2 + x$ at $(3, 12)$ by setting up and evaluating a limit. Show all work, including required comments.

   Use the slope you just computed to find the equation of the tangent line to $y = x^2 + x$ at the point $(3, 12)$. You may give the equation of the tangent line in point-slope or slope-intercept form.

7. Use the definition of limit (yes, with epsilons and deltas!) to write out the meaning of the assertion

$$\lim_{x \to 2} 3x + 1 = 7$$

Prove the resulting statement with epsilons and deltas. You should clearly show the direction of your argument (what follows from what).