Math 170 Test II

Dr. Holmes

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This test will be given from 10:30 am to 11:35 am. You may use a plain scientific calculator with no graphing or symbolic computation functions. Any cell phones must be turned off and out of sight.

The multi-part question with all the derivatives is worth a lot (probably twice as much as one of the other questions).

The other questions all have the same value.

1. State the definition of the derivative $f'(x)$ as a limit.

   Compute the derivative of $f(x) = \sqrt{x}$ using the definition of the derivative as a limit.

   Give an equation for the tangent line to $y = \sqrt{x}$ at $(16, 4)$. Note that you do not need to know how to do the first part to get credit for this!

2. A picture of part of the graph of a function $f(x)$ is shown.

   Identify points on the graph at which $f'(x) = 0$ (these will be among the marked points on the graph).

   Identify points on the graph at which $f'(x)$ does not exist, and at each such point give a brief explanation of why it does not exist (just a phrase). These will be among the marked points on the graph.

   Identify intervals between marked points on which $f'(x)$ is positive and intervals on which $f'(x)$ is negative.

   Identify intervals between marked points on which $f''(x)$ is positive and intervals on which $f''(x)$ is negative.

3. Do one of the two parts. If you do both your best work will count.

   (a) Suppose $y = \sin(u), u = \sqrt{v}, v = x^3 - x$. Compute $\frac{dy}{du}$, $\frac{du}{dv}$ and $\frac{dv}{dx}$, and explain how these can be used to compute $\frac{dy}{dx}$ (this is Leibniz's notation for the chain rule). State your final answer for $\frac{dy}{dx}$ in a form which does not involve any letter but x.

(b) A curve is given by parametric equations $x = t^2$, $y = t^3$. Find the slope of the tangent line to this curve at the point $(4, 8)$ (where $t = 2$) and give an equation for the tangent line.

4. A curve has the equation $x^2 + xy + y^2 = 7$.

   Find $\frac{dy}{dx}$ by implicit differentiation.

   Find an equation for the tangent line to this curve at the point $(1, 2)$.

5. (optional: may replace lowest of the previous problems, but not the following problems) Use implicit differentiation on the formula $x = \tan(y)$ and trigonometric identities to verify that $\frac{d}{dx}[\arctan(x)] = \frac{1}{1+x^2}$.

6. Do one of the two related rates problems. If you do both, your best work will count. If you do very well on both, you might get a little extra credit.

   (a) A woman five feet tall walks at 4 ft/sec toward a spotlight which is on the ground 30 feet from the wall of a tall building. How fast is her shadow moving up the wall when she is 10 feet from the spotlight?

(b) The bow of a boat is 5 feet below the level of a dock. A rope leading to a coil of rope right on the edge of the dock is tied to the bow of the boat. How fast is the rope uncoiling when the horizontal distance from the boat to the dock is 12 feet, if the boat drifts away from the dock at 2 ft/sec?

Make the unrealistic assumption that the rope makes a straight line from the bow of the boat to the dock. We do know it is unrealistic!

7. Lots of derivatives. There is no need to simplify algebraically.

(a)

$$\frac{d}{dx}[5x^3 - 2x^4 + 112x + 14]$$

(b)

$$\frac{d}{dx}[x^3 - \frac{1}{x^2} + \sqrt{x^5}]$$

You should not use the quotient rule or the chain rule here; these are just powers of x.

(c)

$$\frac{d}{dx}[\frac{2x^4 + x^3 + x + 1}{x^2}]$$

Again, there is no need to use the quotient rule here.

(d)

$$\frac{d}{dx}[x^2 \tan(x)]$$

(e)

$$\frac{d}{dx}[\frac{x^2 + 1}{x^3 - x}]$$

(f) Verify using the derivatives of sine and cosine, the quotient rule, and trigonometric identities that $\frac{d}{dx}[\tan(x)] = \sec^2(x)$.

(g)

$$\frac{d}{dx}[e^{2x}\sin(3x)]$$

(h)

$$\frac{d}{dx}[\sqrt{x^3 - x}]$$

(i)

$$\frac{d}{dx}[x \arctan(x^4)]$$

(j)

$$\frac{d}{dx}[\cos(\ln(x^2 - 1))]$$