Math 170 Test III

Dr. Holmes

September 10, 2009

This exam begins at 10:40 and ends at 11:35. You may use a standard scientific calculator without graphing or symbolic computation capabilities. Books, notes and neighbors to remain firmly closed. Cell phones to be turned off and out of sight.

You need only do two of the curve sketching problems (4,5,6). If you do all three your best work will count.

1. Determine the linear function approximating $f(x) = \sqrt{x}$ for x near 16 and use it to estimate $\sqrt{15.8}$.

2. Find the absolute maximum and minimum values of the function $f(x) = \frac{1}{2}x + \sin(x)$ on the interval $[0, \pi]$. Your calculation should include determination of all critical points of the function in the interval.

3. The Mean Value Theorem says that if f is continuous on $[a, b]$ and differentiable on (a, b) then there is a value c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Find a value c satisfying this condition if $f(x) = x^3$ and the interval is $[1, 2]$.

4. Let $f(x) = 2x^3 - 3x^2 - 12x + 2$.

   Determine all critical points of f.

   Identify intervals on which f increases and intervals on which f decreases.

   Classify the critical points of f as relative maxima or minima.

   Determine intervals on which f is concave up and intervals on which f is concave down and identify any points of inflection.

   Use the information to sketch the graph of f in a way that shows all the features you have determined.

5. Let

$$f(x) = \frac{x^2 - 1}{x^2 - 4}$$

Use a sign chart to determine where f is positive and where it is negative.

Identify horizontal and vertical asymptotes of the graph of f.

We supply the information that

$$f'(x) = \frac{-6x}{(x^2 - 4)^2}$$

Identify critical points of f. Use a sign chart to determine where f' is positive and where it is negative (remember that the sign of f' can change at the asymptotes as well as at critical points). State where f is increasing and where f is decreasing.

We supply the information that

$$f''(x) = \frac{6(3x^2 + 4)}{(x^2 - 4)^3}.$$

Make a sign chart for f''. Use it to find out where the graph is concave up and where it is concave down.

Use the information assembled above to sketch the graph of f.

6. We are given the following information about a function f.

$$\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 1$$

$$\lim_{x\to-1-} f(x) = \infty; \lim_{x\to-1+} f(x) = -\infty$$

f is defined, continuous and differentiable everywhere except at $x = -1$

$f'(x) > 0$ for $x < -1$

$f'(x) > 0$ for $0 < x < 1$

$f'(1) = 0; f(1) = 2$

$f'(x) < 0$ for $x > 1$

$f''(x) > 0$ for $x < -1$

$f''(x) < 0$ for $0 < x < 3$

$f''(x) > 0$ for $x > 3$

Write down all information which can be derived from what is given above about vertical and horizontal asymptotes, intervals of increase and decrease of f, and intervals where the graph of f is concave up and down.

Sketch the graph of f. Identify any local maxima or minima, or points of inflection.

7. Do one of the two max/min word problems. If you do both your best work will count. If you do well on both you may earn some extra credit. In each word problem supply a verification that the minimum you find is at least a local minimum.

   (a) A cardboard box with an open top and a base twice as long as it is wide has a volume of 1000 cubic inches. Determine the dimensions for such a box which minimize the amount of cardboard used. State the dimensions to two decimal places.

(b) A poster is to contain 600 square inches of text. The top margin is to be 2 inches; the side and bottom margins are to be one inch. Compute the dimensions for the poster which minimize the total amount of paper used. State the dimensions to two decimal places.

8. Evaluate each of the limits. Use L'Hôpital's Rule if it is appropriate to do so; if it is not appropriate you will be able to evaluate the limit in a different way. Where the rule is appropriate, indicate why it is appropriate.

   You might need to apply the rule more than once.

   (a)

$$\lim_{x \to 0} \frac{\sin(2x)}{\sin(5x)}$$

   (b)

$$\lim_{x \to 0} \frac{\cos(2x)}{\cos(5x)}$$

   (c)

$$\lim_{x \to \infty} \frac{x^2}{e^x}$$

   (d)

$$\lim_{x \to 0+} x \ln(x)$$

   Hint: convert this to a quotient and the rule will apply.