Math 170 Test I

Dr. Holmes

The exam will begin at 9:40 am and end at 10:35 am. You will need a plain scientific calculator without graphing or symbolic computation capabilities. You may not use a cell phone, PDA, or more capable calculator for this purpose. Books, notes and neighbors to remain firmly closed.

Good luck!

1. Estimation of velocity

   Suppose that the position of an object at time t seconds after a fixed starting time is t^3 feet along a straight track from a reference point.

   Determine the average velocity of the object during the interval of time $[2, 2.1]$. Determine the average velocity of the object during the interval of time $[1.9, 2]$.

   Compute some average velocities over "better" intervals (tell me which ones) and estimate the instantaneous velocity of the object at $t = 2$. Use at least two "better" intervals and don't neglect the time before $t = 2$ or the time after it.

2. Limits

   Estimate the limit of $\lim_{t \to 0} \frac{\sin(t)}{t}$ by computing relevant values of this function with your calculator.

   Do we have any way to evaluate this limit exactly using the limit laws? Explain.

   Compute the following limits. You do not need to use epsilons and deltas, nor do you need to do complete step-by-step application of the limit laws, but you do need to show work (including required comments).

   (a)
   $$\lim_{x \to 3} \frac{x - 3}{x^2 - 9}$$

   (b)
   $$\lim_{x \to 2} \frac{1}{\sqrt{x + 2} + \sqrt{4}}$$

   (c)
   $$\lim_{x \to \infty} \frac{1 + 3x^2}{1 - 2x^2}$$

3. Express the assertion that $f(x) = \frac{1}{x+1}$ is continuous at 2 using the definition of continuity: this will be an equation involving a limit.

   Then verify the correctness of this equation using step-by-step application of the limit laws (this does not involve any epsilons or deltas!)

4. State the limits at infinity or infinite limits (including one-sided and two-sided infinite limits) of the function f whose graph is shown. Hint: an infinite limit or limit at infinity is associated with each asymptote pictured.

5. The graph of a function f is shown. Identify the x values at which f is discontinuous. State what kind of discontinuity there is at each of these values. Identify any of these values at which the function is continuous from the left or right, and list the longest intervals on which the function is continuous.

   Sketch the graph of the derivative of f (right on this picture – don't try to make a separate graph). Your graph doesn't need to be incredibly precise, but it should show where f' is zero, where it is positive and where it is negative, and where it is larger or smaller.

6. Compute the derivative of $f(x) = x^2 + x$ using the definition of the derivative as a limit, as follows:

   State the general definition of the derivative as a limit for any function f (I do not care which of the two forms you use).

   Set up and evaluate this limit for $f(x) = x^2 + x$.

   Tell me what function is the derivative of $x^2 + x$.

   Use the derivative to find the equation of the tangent line to $y = x^2 + x$ at the point $(3, 12)$. You may give the equation of the tangent line in point-slope or slope-intercept form. The slope of this tangent line is a constant: it will not have any x's in it!

7. Use the definition of limit (yes, with epsilons and deltas!) to write out the meaning of the assertion

$$\lim_{x \to 2} 3x + 1 = 7$$

Prove the resulting statement with epsilons and deltas. You should clearly show the direction of your argument (what follows from what).