Math 170 Test I

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This test will begin at 9:40 and end at 11:20. You may use a plain scientific calculator ; you may not use graphing calculators, nor may you use a cell-phone or PDA as a calculator. You may not use books, notes or neighbors. You must show all work and certainly any specific calculations I request for full credit.

1. The position of a particle at time $t > 0$ is given by $P(t) = \frac{1}{t+2}$.

   Determine the average velocity of the particle on each of the following time intervals (positive or negative sign does matter; set up your calculation on your paper and use as many places of accuracy as your calculator shows):

   (a) $[0.9, 1]$

   (b) $[1, 1.1]$

   Use the results of the calculations above to guess at the instantaneous velocity of the particle at time $t = 1$.

   Set up and compute a better estimate (still in the form of an average velocity).

2. Give the appropriate answer to each limit question, either a definite value, or ∞, or $-\infty$, or "does not exist". In each case, you must show appropriate calculation and/or explanation to support your answer.

   (a)

$$\lim_{x \to 3} \frac{x-3}{x^2-9}$$

(b)

$$\lim_{x \to 2} \frac{x+2}{x^2+4}$$

(c)

$$\lim_{x \to 2-} \frac{x-4}{x^2-4}$$

3. Verify that $f(x) = x^2 + 4$ is continuous at 4 using the definition of continuity and the limit laws. That is, express the statement that this function is continuous at 4 as a statement about limits, then prove that the statement is true using the limit laws (not using ϵ's or δ's) and showing *all* steps.

4. Sketch the graph of a function f which satisfies

$$\lim_{x \to 0} f(x) = -\infty,$$

$$\lim_{x \to \infty} f(x) = 2,$$

$$\lim_{x \to -\infty} f(x) = -2,$$

$$\lim_{x \to 2-} = 1,$$

$$\lim_{x \to 2+} = 0,$$

   is continuous everywhere except at 0 and 2 and continuous from the left at 2.

5. A graph is given with a number of labelled values on the x-axis. Identify those values at which the function fails to be continuous, and at such values write such statements about the limit of the function or its limits from the left and right as are appropriate.

   Identify those labelled values at which the function fails to have a derivative and state the reason why it fails to have a derivative at each of these points.

6. Set up and evaluate a limit (showing all work) representing the slope of the tangent line to $y = \sqrt{x+2}$ at the point $(7, 3)$.

   Determine an equation of the tangent line.

7. Determine the derivative $f'(x)$ of the function $f(x) = \frac{1}{x}$, using the definition of the derivative as a limit, and use the result to write an equation for the tangent line to this curve at $(\frac{1}{3}, 3)$.

   Suppose that the tangent line to the graph of a function g at the origin passes through the point $(2, 3)$. What is $g'(0)$?

8. Use the official definition of limit to state the meaning of

$$\lim_{x \to 2} 2x + 1 = 5$$

   (you will get a sentence with ϵ's and δ's) and prove the resulting statement.