

# Math 170 Test 2

Dr. Holmes

July 2, 2002

The test starts at 9:35 am and ends at 11:35 am. You may not use any calculator with graphing or symbolic calculation capabilities. There is some use for a plain scientific calculator. Show all work!

Happy Independence Day!

1. Evaluate the indicated derivatives. There is no need to simplify past the point where all derivatives have been computed.

(a)  $\frac{dy}{dx}$  when

$$y = 2x^3 + 3x^2 + 6$$

(b)  $\frac{dy}{dx}$  when

$$y = 5x^4 + \frac{6}{x^2} - \sqrt[3]{x^2}.$$

(c)  $f'(x)$  when

$$f(x) = (\sqrt{x} + 1) \cos(x)$$

(d)  $g'(x)$  when

$$g(x) = \frac{\sin(x)}{\ln(x)}$$

(e)  $h'(x)$  when

$$h(x) = \ln(\cos(x))$$

(f)  $k'(x)$  when

$$k(x) = (x \tan(x))^7$$

(g)

$$\frac{d}{dx} \left[ \frac{x^2 - 1}{x^3 + 1} \right]$$

(h)

$$\frac{d}{dx} [\ln(\arctan(x^2))]$$

2. Suppose we have the following information:

$$f(1) = 2; g(1) = 3; f'(1) = -1; g'(1) = 5; f(2) = 6; g(2) = -4; f'(2) = 3; g'(2) = -2$$

(a) Evaluate the derivative

$$\frac{d}{dx}[f(x)g(x)]$$

at  $x = 2$ .

(b) Evaluate the derivative

$$\frac{d}{dx}[g(f(x))]$$

at  $x = 1$ .

(c) Evaluate the derivative

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]$$

at  $x = 1$ .

3. A manufacturer produces widgets. The profit from production of  $x$  widgets is

$$P(x) = 25000 + 6000x - 5x^2.$$

- (a) Compute the marginal profit function.
- (b) Evaluate the marginal profit function at  $x = 100$ . What quantity does this estimate that we can write down using the profit function without calculus?
- (c) What advice about level of production would you give to the manufacturer after evaluating the marginal profit function at  $x = 100$  (assuming that he is planning to produce 100 widgets)? What advice would you give if the manufacturer was planning to produce 700 widgets, on the basis of the value of the marginal profit function at  $x = 700$ ? Hint: consider the sign of the marginal profit.

4. Compute the derivative of  $y = \cos(\ln(\sqrt{x^2 - 1}))$  using the chain rule in the form

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dv} \frac{dv}{dw} \frac{dw}{dx}.$$

Show all work.

5. In the following formula,  $y$  stands for an unknown function of  $x$ . Use implicit differentiation to find  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ .

$$4y^2 - 6xy^2 + x^3 = 0$$

Give an equation for the tangent line to this curve at the point  $(2, 1)$ .

6. Compute the second derivative of each function.

(a)

$$f(x) = 3x^5 - 5x^3$$

(b)

$$f(x) = \ln(x)$$

(c)

$$f(x) = \tan(x)$$

7. Related rates problems. You may choose one. If you do both parts, the best one will count.

- (a) The deck of a boat is level with the dock. A rope passes from the prow of the boat over a pulley 10 feet above the end of the dock. The boat is drifting away from the dock at 0.5 miles per hour. The rope pays out freely from a coil of rope on the deck as the boat moves away. How fast is the rope paying out when the boat is 8 feet from the dock?

You may assume (unrealistically) that the length of rope from the pulley to the prow of the boat is a straight line.

- (b) A searchlight rests on the ground, 15 feet from the wall of a tall building. A six-foot tall man walks toward the searchlight at 3 feet per second. How fast is his shadow moving up the wall when he is 10 feet from the searchlight?