Math 170 Test II

Dr. Holmes

February 24, 2005

The test begins at 9:40 am and ends at 10:35 pm. Please leave the class promptly if your exam is collected at 10:35, so that the next class can get in efficiently.

You may use a plain scientific calculator without graphing or symbolic computation capabilities. Other calculators may not be used. Books, notes, and neighbors are to remain firmly closed.

Please check that your paper is complete, with 6 questions on 9 numbered pages. The weight of problem 2 is considerable (possibly up to half of the exam), while the weights of the other problems are equal and smaller. It is not worth trying for extra credit opportunities unless you have finished the rest of the test.

1. Tangent lines and derivatives:

   (a) Find an equation of the tangent line to

   $$y = x^2 + \sqrt{x}$$

   at $(4, 18)$.

   (b) At what points does the graph of

   $$y = x^3 - 27x + 5$$

   have horizontal tangent lines? (Your answer(s) really should be a point or points, with x and y coordinates).

2. Find the indicated derivatives. No simplification is required unless specifically asked for. I strongly recommend bringing constant coefficients (especially negative ones) to the fronts of terms. Do not omit needed parentheses.

(a)

$$\frac{d}{dx}[5x^4 + 3x^2 + 114x - 23]$$

(b)

$$\frac{d}{dx}[\frac{x^3 + x - 1}{\sqrt{x}}]$$

Find this derivative *without* using the Quotient Rule or the Product Rule.

(c)

$$\frac{d}{dx}[\frac{1 + x^2}{1 - x^2}]$$

Use the Quotient Rule and simplify the numerator.

(d)

$$\frac{d}{dx}[e^x \cos(x)]$$

(e)

$$\frac{d}{dx}[(x^3 - 5x)^{\frac{3}{2}}]$$

(f)

$$\frac{d}{dx}[\sin(\sqrt{1 - x^2})]$$

(g)

$$\frac{d}{dx}[x^2 e^{-8x}]$$

Extra credit (small) if you can tell me at what values of x this graph has horizontal tangents.

(h)

$$\frac{d}{dx}[x \arctan(2x)]$$

(i)

$$\frac{d}{dx}[x\tan(x^2)]$$

(j)

$$\frac{d}{dx}[x\sin^2(x)]$$

3. The graphs of functions f and g are shown. What appear to be straight line segments really are straight line segments. Using the graph, determine the following values of derivatives. Show your work in enough detail that I can tell how you got your answers.

   (a) $u'(1)$, where $u(x) = f(x)g(x)$

   (b) $v'(1)$, where $v = f(g(x))$

4. If $y = \tan(u)$, $u = \sqrt{v}$, and $v = 1 + x^3$, determine $\frac{dy}{dx}$ using the chain rule in the form

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dv}\frac{dv}{dx}.$$

The final result should be stated entirely in terms of x.

You must show the complete setup for this particular method and all steps of your work (just writing the derivative does not carry much credit).

5. Determine an equation for the tangent line to the graph of

$$x^2 + 2xy^2 + y^4 = 9$$

at the point $(2, 1)$ (by implicit differentiation).

6. Proofs and derivations. Do your choice of the following. If you do both, you will receive credit for your best work. Some extra credit may accrue if you do well on both.

   (a) Prove the Reciprocal Rule for derivatives

   $$\frac{d}{dx}\left[\frac{1}{f(x)}\right] = -\frac{f'(x)}{(f(x))^2}$$

   using the definition of the derivative as a limit.

   (b) Find $\frac{d}{dx}[\arctan(x)]$ by applying implicit differentiation to the formula $x = \tan(y)$. Show all work. Just writing down the derivative of the arc tangent function carries no credit.