Math 170 Test Two

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This exam will start at 9:40 am and end at 11:25 am.

You may not use any calculator with graphing or symbolic computation capability. There may be some use for a plain scientific calculator.

Show all work. It is not necessary to simplify algebraically past the point at which all derivatives have been taken unless I tell you explicitly to simplify.

1. Compute all derivatives. It is not necessary to simplify unless you are explicitly told to do this.

   (a)
   $$\frac{d}{dx}[4x^5 + 5x^4 + 2x^3 - 15x - 11]$$

   (b)
   $$\frac{d}{dx}\left[\frac{x + \sqrt{x}}{x^5}\right]$$

   (c)
   $$\frac{d}{dx}\left[\frac{A}{x} + \frac{B}{\sqrt{x}} + \frac{C}{x^3}\right]$$

(d)

$$\frac{d}{dx}\left[\frac{x^2-1}{x^3+2x}\right]$$

(e)

$$\frac{d}{dx}[\cot(x)] = \frac{d}{dx}\left[\frac{\cos(x)}{\sin(x)}\right]$$

Derive this from the derivatives of the sine and cosine, rules of differentiation, and basic trig identities – you are expected to simplify, and there is no credit for just remembering it.

(f)

$$\frac{d}{dx}[(x^3+2x)^7(x^2-1)^4]$$

Please don't multiply anything out!

(g)

$$\frac{d}{dx}[\sin(\sqrt{x^3-1})]$$

(h)

$$\frac{d}{dx}\left[\left(\frac{x-1}{x^2+1}\right)^3\right]$$

Use logarithmic differentiation; don't simplify algebraically.

(i)

$$\frac{d}{dx}[\arctan(\ln(x))]$$

2. The graphs of two functions f and g are given below. Pieces of the graphs that appear to be straight line segments really are straight line segments.

   Evaluate the following quantities:

   (a) $h'(2)$, where $h(x) = f(x)g(x)$.

   (b) $j'(2)$, where $j(x) = f(g(x))$.

3. The quantity y is implicitly defined as a function of x by the equation

$$2x^2 + xy - y^3 + 4 = 0.$$

Determine $\frac{dy}{dx}$ (in terms of x and y) by implicit differentiation.

Give an equation for the slope of the tangent line to this curve at the point $(1, 2)$.

4. Find the derivative of the arc tangent function

$$y = \arctan(x)$$

by implicit differentiation of the equation $x = \tan(y)$. Your final answer must be stated entirely in terms of x.

5. Find the derivative $\frac{dy}{dx}$ of

$$y = \sin(\sqrt{\ln(x)})$$

using the Chain Rule in the form

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dv}\frac{dv}{dx}.$$

Your work must define appropriate quantities u and v and use them, but your final result needs to be in terms of x alone.

6. From a picture, label the graphed functions as f, f', and f''. Explain.

7. Determine the requested higher derivatives.

(a)

$$\frac{d^3}{dx^3}[x^5]$$

(b)

$$\frac{d^2}{dx^2}\left[\frac{x-1}{x+1}\right].$$

Simplify.

(c)

$$\frac{d^{100}}{dx^{100}}[\sin(x)].$$

Explain.

8. Choose one of the two problems. If you do both, your best work will count.

   (a) A telescoping ladder leans against the side of a building. The base of the ladder is immobilized by a cinder block 3 feet from the wall of the building, but the ladder begins to collapse, getting shorter at a rate of 3 inches per second. How fast is the top of the ladder sliding down the side of the building when it is 4 feet from the ground?

   Hint: please be careful about units!

(b) A woman who is five feet tall is walking away from a building toward a searchlight which is on the ground twelve feet from the side of the building. She walks at two feet per second. How far is she from the searchlight when her shadow is moving up the side of the building at four feet per second?