Math 170, Test III

M. Randall Holmes

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This exam will begin at 7:40 am and end at 9:35 am. You are not permitted to use calculators with graphing or symbolic computation capabilities. You may have some use for a plain scientific calculator, but as a rule exact answers are expected (except in the case of approximate calculations as in word problems).

1. Find the absolute maximum and minimum values of the given function on the given interval. Answers must be exact (no calculator output), though you might use your calculator to determine which values are largest or smallest.

   (a)
   $$x^3 - 9x + 5, [1, 3]$$

   (b)
   $$x \ln(x), [\frac{1}{3}, 3]$$

2. The conditions of the Mean Value Theorem apply to one of the functions listed below on the associated interval, and do not apply to the other. Identify the function and interval to which the theorem does not apply, and explain why not. For the other function, find a value of c satisfying the conclusion of the theorem.

   (a)
   $$x^{2/3}, [-2, 1]$$

   (b)
   $$x^3, [2, 5]$$

3. Sketching a cubic curve.

   For the following function, use calculus to determine all intervals of increase and decrease, local maxima and minima, intervals of concavity up and down and points of inflection. Then sketch the graph of the function, labelling all significant points with x and y coordinates. Show all work supporting your results.

$$f(x) = x^3 - 3x^2 - 9x + 1$$

4. Sketching the graph of a rational function, given its derivatives.

   A function and its first and second derivatives are given. Use this information to determine all intercepts, asymptotes, intervals of increase and decrease, local maxima and minima, intervals of concavity up and down, and points of inflection. Then sketch the graph of the function, labelling all significant points with x and y coordinates. Show all work supporting your results.

$$f(x) = \frac{x}{(x+2)^2}$$

$$f'(x) = \frac{2-x}{(x+2)^3}$$

$$f''(x) = \frac{2x-8}{(x+2)^4}$$

5. Evaluate the following limits, using L'Hôpital's Rule if it applies. Your work should indicate clearly why application of the rule is appropriate when you use it, and why it is not appropriate when you do not use it. Note that you are asked to evaluate the limit even if the rule does not apply.

(a)

$$\lim_{x \to 0} \frac{\cos(x) - 1}{\sin(x)}$$

(b)

$$\lim_{x \to 0} \frac{x}{x^2 + 2x + 4}$$

(c)

$$\lim_{x \to \infty} \frac{x^2}{e^{2x}}$$

6. Related rates

   A ladder 13 feet long is leaning against the side of a building. The end of the ladder is sliding away from the side of the building at 3 ft/sec. How fast is the top of the ladder sliding down the wall when the foot of the ladder is five feet from the building?

7. Do one of the following word problems. If you complete more than one problem, your best work will count.

   (a) A square sheet of cardboard 5 ft by 8 ft is to have squares of equal size cut out of each corner as shown and the side flaps folded up to form a box. What are the dimensions of the box of largest volume that can be made in this way?

(b) An enterprising ten-year old decides to sell lemonade on the steps of the State Capitol on hot summer days. She finds that she can sell 1000 cups of lemonade per day at 25 cents per cup. For each cent that she raises the price, she loses 100 customers (and for each cent that she lowers the price, she gains one hundred customers). It costs her one cent to make each cup of lemonade. How many cups of lemonade should she sell at what price for maximum profit?

You do not need to set up your equations in terms of the number of cups sold – you may choose to work in terms of some other quantity in the problem. (I suggest the number of cents that she raises the price.)