Math 170 Test III

Dr. Holmes

The test will begin at 9:40 am and end at 10:35 am. You are permitted to use a plain scientific calculator with no graphing or symbolic computation capability: no more competent calculators, no PDA's and no cell phones. Cell phones must be turned off and out of sight. Books, notes and neighbors to remain firmly closed.

1. Second derivatives and logarithmic differentiation.

   (a) Compute the second derivative of $\tan(x)$.

   (b) Compute

   $$\frac{d}{dx}[\frac{\sqrt{x-1}}{(x+1)^3}]$$

   using logarithmic differentiation.

2. Estimate $\sqrt{15.9}$ by using a linear approximation to the function $f(x) = \sqrt{x}$ near $x = 16$, or (equivalently) by using differentials.

3. Find the absolute maximum and minimum values of the given functions on the given intervals. Values must be exact (no calculator output), though you are permitted to use your calculator to determine which values are largest and smallest.

   (a)
   $$x^3 - 6x + 5, [1, 3]$$

   (b)
   $$\sin(x) + \cos(x), [0, \frac{\pi}{2}]$$

4. The conditions of the Mean Value Theorem apply to one of the functions listed below on the associated interval, and do not apply to the other. Identify the function and interval to which the theorem does not apply, and explain why not. For the other function, find a value of c satisfying the conclusion of the theorem.

   (a)
   $$|x|, [-1, 2]$$

   (b)
   $$x^{3/2}, [1, 4]$$

5. For the following function, use calculus to determine all intervals of increase and decrease, local maxima and minima, intervals of concavity up and down and points of inflection. Then sketch the graph of the function, labelling all significant points with x and y coordinates. Show all work supporting your results.

$$f(x) = 2x^3 - 3x^2 + 12x + 5$$

6. Evaluate the following limits, using L'Hôpital's Rule if it applies. Your work should indicate clearly why application of the rule is appropriate when you use it, and why it is not appropriate when you do not use it.

(a)

$$\lim_{x\to\infty} \frac{x^2}{e^x}$$

(b)

$$\lim_{x\to\pi} \frac{\sin(x)}{1-\cos(x)}$$

(c)

$$\lim_{x\to 0} x\cot(2x)$$

Hint: convert to a quotient.

7. Related rates problems. You may choose one. If you do both parts, the best one will count.

   (a) The deck of a boat is level with the dock. A rope passes from the prow of the boat over a pulley 8 feet above the end of the dock. The boat is drifting away from the dock at 3 feet per second. The rope pays out freely from a coil of rope on the deck as the boat moves away. How fast is the rope paying out when the boat is 10 feet from the dock?

   You may assume (unrealistically) that the length of rope from the pulley to the prow of the boat is a straight line. There are no tides.

   Give your answer to as many decimal places of accuracy as your calculator provides.

(b) A searchlight rests on the ground, 12 feet from the wall of a tall building. A five-foot tall woman walks toward the searchlight at 4 feet per second. How fast is her shadow moving up the wall when she is 6 feet from the searchlight?