Math 170, Test III

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This exam will begin at 9:40 am and end at 11:25 am. You are not permitted to use calculators with graphing or symbolic computation capabilities. You may have some use for a plain scientific calculator, but as a rule exact answers are expected (except in the case of approximate calculations as in word problems or Newton's method).

1. Find the absolute maximum and minimum values of the given functions on the given intervals. Values must be exact (no calculator output – but you might want to use a calculator to determine whether one exact value is larger than another).

   (a)
   $$x^3 - 12x + 5, [1, 3]$$

   (b)
   $$x \ln(x), [\frac{1}{3}, 3]$$

2. The conditions of the Mean Value Theorem apply to one of the functions listed below on the associated interval, and do not apply to the other. Identify the function and interval to which the theorem does not apply, and explain why not. For the other function, find a value of c satisfying the conclusion of the theorem.

   (a)
   $$x^{2/3}, [-2, 1]$$

   (b)
   $$x^2, [1, 4]$$

3. For the following function, use calculus to determine all intervals of increase and decrease, local maxima and minima, intervals of concavity up and down and points of inflection. Then sketch the graph of the function, labelling all significant points with x and y coordinates. Show all work supporting your results.

$$f(x) = x^3 - 3x^2 - 9x + 1$$

4. A function and its first and second derivatives are given. Use this information to determine all intercepts, asymptotes, intervals of increase and decrease, local maxima and minima, intervals of concavity up and down, and points of inflection. Then sketch the graph of the function, labelling all significant points with x and y coordinates. Show all work supporting your results.

$$f(x) = \frac{x}{(x+2)^2}$$

$$f'(x) = \frac{2-x}{(x+2)^3}$$

$$f''(x) = \frac{2x-8}{(x+2)^4}$$

5. Evaluate the following limits, using L'Hôpital's Rule if it applies. Your work should indicate clearly why application of the rule is appropriate when you use it, and why it is not appropriate when you do not use it. Give answers of ∞ or $-\infty$ (when appropriate) in preference to "undefined".

   (a)
   $$\lim_{x\to 0+} \frac{\ln(x)}{x}$$

   (b)
   $$\lim_{x\to 0} \frac{\sin(x)}{\cos(x) - x - 1}$$

   (c)
   $$\lim_{x\to \frac{\pi}{2}} (x - \frac{\pi}{2}) \sec(x)$$

6. Estimate the square root of 5 to four decimal places of accuracy by applying Newton's Method to solve the equation $x^2 - 5 = 0$. Show all work. Simply computing the square root of 5 on your calculator carries no credit, though of course you can do this as a check.

7. Do one of the following word problems. If you complete both problems, your best work will count. You must show all work.

   (a) A square sheet of cardboard 5 ft by 8 ft is to have squares of equal size cut out of each corner as shown and the side flaps folded up to form a box. What are the dimensions of the box of largest volume that can be made in this way?

(b) An enterprising ten-year old decides to sell lemonade at the entrance to the county fairgrounds on hot summer days. She finds that she can sell 2000 cups of lemonade per day at 10 cents per cup. For each cent that she raises the price, she loses 50 customers (and for each cent that she lowers the price, she gains fifty customers). It costs her two cents to make each cup of lemonade. How many cups of lemonade should she sell at what price for maximum profit?

You do not need to set up your equations in terms of the number of cups sold – you may choose to work in terms of some other quantity in the problem. (I suggest the number of cents that she raises the price.)