Math 175 Test III

Dr. Holmes

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This exam begins at 11:40 and ends at 12:35. You may use a standard scientific calculator without graphing or symbolic computation capabilities. Cell phones must be turned off and out of sight.
1. Determine the antiderivative using an appropriate trigonometric substitution.

\[ \int \sqrt{1 - x^2} \, dx \]
2. Compute the following integrals using the method of partial fractions. Show all supporting work.

(a) \[ \int \frac{2x - 1}{(x - 1)(x - 3)} \, dx \]

(b) \[ \int \frac{1}{x(x^2 + 1)} \, dx \]
3. Set up the sum estimating
\[ \int_{1}^{5} \frac{1}{x} \, dx \]
using Simpson’s Rule with four partitions. It is sufficient to give the setup; you do not need to compute it.

How many partitions are needed to estimate this integral with error less than 0.0001 using Simpson’s Rule? I remind you that the error bound formula is
\[ E_S \leq \frac{M(b - a)^5}{180n^4} \]
where \( n \) is the number of partitions and \( M \) is an upper bound for the absolute value of the fourth derivative of the integrated function on the interval of integration. You do need to actually compute the number of partitions in this part.
4. Set up each of the following improper integrals as limits of definite integrals. Compute one of them.

(a) \[ \int_0^4 \frac{1}{\sqrt{x}} \, dx \]

(b) \[ \int_1^\infty \frac{1}{x^4} \, dx \]
5. Determine whether \[ \int_{2}^{\infty} \frac{1}{\sqrt{x}} \, dx \] converges or diverges by comparison with an integral of the form \[ \int_{2}^{\infty} \frac{1}{x^p} \, dx. \] Give all information that you use and explain how you use it.
6. Sequences

(a) Compute the first six terms of the sequence with the following definitions: \( a_1 = 2; a_2 = 1; a_n = 2a_{n-2} + a_{n-1} \) for each \( n > 2 \).

(b) Determine the limit \( \lim_{n \to \infty} \ln(2n - 1) - \ln(3n + 2) \). Show all work.
7. Series

(a) Compute the exact value of the sum

\[ \sum_{n=0}^{\infty} \left( \frac{2}{3^n} + \frac{3}{2^n} \right). \]

Show appropriate work.

(b) Compute the exact value which the sum

\[ \sum_{n=0}^{\infty} [x(x - 2)^n] \]

takes when it converges and state the range of values of \( x \) for which the series converges.