Math 175 Test I

Dr. Holmes

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This exam will begin at 11:40 am and end at 12:35 pm (halfway through the break between classes). You are permitted to use a scientific calculator without graphing or symbolic computation capability. Please keep your calculator visible on your desk without its cover so I can check it easily. Cell phones must be turned off and out of sight.

Except in problem 7, give exact answers.

Good luck!
1. Determine the average value of the function $y = x^2$ on the interval $[1, 3]$

Find a value in $[1, 3]$ at which the function takes on this average value.
2. Do one of the two area problems. Doing both may be good for some additional credit (don’t try for additional credit before finishing the exam!)

(a) Sketch the region bounded by \( y = 2.5 - x \) and \( y = \frac{1}{x} \) and determine its area.
(b) Sketch the regions bounded by $y = x$, $y = x^3$, $x = 0$, and $x = 2$.

The two graphs do cross! Determine the total area of the regions.
3. Sketch the region in the first quadrant bounded by $y = 2 - x$, $y = x^2$, and the $y$-axis. Determine the volume of the solid obtained by revolving this region around the $x$-axis, using the method of disks and washers.
4. Sketch the region in the first quadrant bounded by $y = 4 - x^2$, the $x$-axis and the $y$-axis. Determine the volume of the solid obtained by revolving this region around the $y$-axis, using the method of cylindrical shells.

Set up but do not evaluate the integral needed to determine the volume of the solid obtained by revolving this region around the line $x = 2$ (using cylindrical shells).
5. Estimate

\[ \int_{1}^{3} \sqrt{x} \, dx \]

using the Trapezoid Rule with \( n = 4 \). How many subdivisions would be needed to guarantee accuracy within .01? (the bound on error is \( \frac{K(b-a)^3}{12n^2} \), where \( K \) is larger than \( |f''(x)| \) for any \( x \) in \([a,b]\): the correct value of \( K \) is fairly easy to determine here, but I tell you for free that you can use \( K = 1 \) for a slight point penalty).