Math 175, Test I

Dr. Holmes

February 7, 2002

This exam begins at 9:40 am and ends at 10:35 am.

Calculators with graphing or symbolic computation capability are not allowed. You should not use a calculator on this exam except to do addition, subtraction, or multiplication of whole numbers (or division if there is no remainder). *Exact* answers are required in all problems (except the Simpson's rule calculations in the last problem), not calculator approximations (you may give a calculator estimate in addition to an exact answer – this would be appropriate in the work problem – but not in place of it). Also, any calculation you do with your calculator should be set up on your paper so I can see what you did.

1. Determine the area of the region bounded by $x = 0, x = 5, y = x^2$ and $y = x^3$. Be sure to draw a picture of the region. Hint: the curves do cross!

2. Determine the average value of the function $f(x) = x^2$ on the interval $[-5, -1]$.

   Find a number c between -5 and -1 such that the value of the function at c is equal to the average value of the function on the interval.

3. Sketch the region bounded by the curve $y = 9 - x^2$ and the x-axis and sketch a typical "slice" of the solid obtained by revolving this region around the x-axis. Set up and evaluate an integral which will give the volume of this solid.

Set up but do not evaluate the integral which gives the volume of the solid obtained by rotating the same region around the y-axis, using the method of disks and washers (*not* the method of cylindrical shells). There is something mildly tricky about this!

4. Sketch the region bounded by the curve $y = 4x - x^2$ and the x-axis.

Determine the volume of the solid obtained by rotating this region around the y-axis by the method of cylindrical shells: sketch a typical cylindrical shell, indicating its radius and height (you can do this on your sketch of the region), then set up and evaluate the integral which gives the volume.

Set up but do not evaluate the integral which gives the volume of the solid obtained by rotating the same region around the line $x = -3$.

5. Do one of the two problems. If you do both, the best one will be graded, and good performance on both will be good for extra credit.

   (a) A rope 100 feet long is suspended from the top of a sufficiently tall building. The rope weighs one pound per foot. At the end of the rope is a 15 pound bucket which starts with 70 pounds of sand in it, which is leaking: it leaks a pound of sand for each foot it is drawn upward. How much work is done (on bucket, rope, and sand) in hauling the bucket to the top of the building?

(b) A pit in the ground takes the form of an inverted square pyramid, 100 feet deep in the center and 300 feet on a side at ground level. The pit is entirely filled with sand, which for purposes of this problem weighs 150 pounds per cubic foot.

The bed of one of our monster sand trucks is ten feet above the ground. Set up an integral which determines the amount of work in foot-pounds required to empty all the sand in the pit into trucks. You do not need to evaluate the integral.

6. (a) Estimate

$$\arctan(2) = \int_0^2 \frac{1}{1+x^2}\,dx$$

using Simpson's rule with four subdivisions. Show your setup clearly. Please do your calculations to three or four decimal places.

(b) How many subdivisions would you need to estimate

$$\ln(4) = \int_1^4 \frac{1}{x}\,dx$$

within .00001 using Simpson's Rule? The bound on the error in the estimate of the integral

$$\int_a^b f(x)\,dx$$

using n subdivisions is

$$\frac{K(b-a)^5}{180n^4},$$

where K is an upper bound on the absolute value of the fourth derivative of $f(x)$. I will remind you for free that the fourth derivative of $\frac{1}{x}$ is $\frac{24}{x^5}$.