Math 187 Truth Table and Logical Notation Problems

Dr. Holmes

1. Verify that $\neg(P \vee Q)$ and $\neg P \wedge \neg Q$ are logically equivalent, using a truth table. Be sure to highlight the appropriate bits of the truth table which show that this is correct.

2. Verify that $(P \wedge Q) \rightarrow R$ and $(P \rightarrow R) \vee (Q \rightarrow R)$ are logically equivalent, using a truth table. Be sure to highlight the appropriate bits of the truth table which show that this is correct.

3. Convert $\neg(P \vee (\neg Q \wedge R))$ to a form in which negation ($\neg$) is only applied to letters, by repeated applications of de Morgan's laws. Show steps (this is basically a calculation).

4. Use the method shown in class to find an expression in $\neg$, $\wedge$, $\vee$ (not, and, or) which has the truth table given below.

P	Q	R	???
T	T	T	F
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	T

   Use the same method to develop an expression for $A \rightarrow B$ from the truth table for implication. Notice that this is not nearly as nice as the expression $\neg A \vee B$ that you have already seen.

5. We introduce the notation $P \downarrow Q$ for $\neg(P \wedge Q)$ (NAND). We showed in class that every expression with a truth table can be expressed in terms of $\vee, \wedge, \neg$, then that $\vee$ can be expressed in terms of $\wedge$ and $\neg$, and then that $\wedge$ and $\neg$ can both be expressed in terms of $\downarrow$ (NAND). So everything can be expressed in terms of NAND.

   We introduce the notation $P|Q$ for $\neg(P \vee Q)$. This is also called NOR, and $P|Q$ can be read "Neither P nor Q". We will show that everything with a truth table can be expressed in terms of NOR.

   (a) Express $\wedge$ in terms of $\vee$ and $\neg$ (Hint: use de Morgan's laws just as I did to express $\vee$ in terms of $\wedge$ and $\neg$).

   (b) Express $\neg P$ using NOR.

   (c) Express $P \vee Q$ in terms of NOR (both of these will be very like what we did with NAND).

   (d) Express $P \to Q$ in terms of NAND alone, and then express it in terms of NOR alone.