

Math 187 Final Exam

Dr. Holmes

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This exam begins at 7:50 am and ends at 9:30 am as usual. You may use your book, but no notes or handouts. You may use a plain scientific calculator without graphing or symbolic computation capabilities. Good luck!

1. Show that an implication $P \rightarrow Q$ is logically equivalent to its contrapositive $\neg Q \rightarrow \neg P$ and *not* logically equivalent to its converse $Q \rightarrow P$ using truth tables.

Be sure to highlight significant columns or rows in your truth table and explain briefly in English what is significant about them or about the relationship between them.

2. Counting problems. In all parts, I want actual numerical answers computed, though it is useful to give a setup in terms of addition, multiplication, and binomial coefficients.

You may do three of the four parts; if you do all four, the best three will count.

- (a) A state has license plates consisting of three letters followed by four digits. If letters may not appear more than once but digits may be repeated freely, how many possible plates are there?

- (b) A subcommittee is to be formed from a committee with 25 members, 20 of whom are men and 15 of whom are women. If the subcommittee is to have 6 members, including a chair and a secretary, how many subcommittees are possible?

If the subcommittee is to have three men and three women, with the chair a woman and the secretary a man, how many subcommittees are possible?

(c) How many one-to-one functions are there from $\{1, 2, 3, 4\}$ to $\{5, 6, 7, 8, 9, 10\}$?

(d) A necklace is to be made out of red, green, blue, yellow and pink beads. The necklace will have 20 beads. We have an unlimited supply of beads of each color. There is a clasp on the necklace past which beads cannot be moved. How many necklaces are possible?
Hint: if you know what you are doing, you can write a single binomial coefficient as the answer (and evaluate it, of course).

3. In a group of 16 friends, 12 take English, 7 take French, and 8 take math. 4 take English and French, 5 take English and math, and just one brave soul takes French and math. How many of the friends take all three subjects?

Your calculation must use the inclusion/exclusion principle (so I should be able to see in your work that you know the form of this principle for three sets).

4. Prove that the relation $a \equiv b \pmod{7}$ is an equivalence relation. Your proof should make it clear that you know the definition of $a \equiv b \pmod{n}$ and that you know the names and the definitions of the three properties that an equivalence relation is required to have.

List the equivalence classes under this relation, showing at least 3 elements of each of the classes.

5. Prove by mathematical induction that for any natural number n , the sum $1 + 6 + 36 + \dots + 6^n$ is equal to $\frac{6^{n+1}-1}{5}$.

6. Compute $\gcd(38, 71)$ using the Euclidean algorithm.

Express $\gcd(38, 71)$ in the form $38x + 71y$ where x and y are integers.

Solve the equation $38z \equiv 21 \pmod{71}$ for z : you can use the first part of the question to solve this (be careful about signs!)

7. Do one of the two parts. If you do both, your best work will count.

- (a) Solve the system of two equations (describe all integer solutions or state a modulus in which you need to work).

$$x \equiv 11 \pmod{23}$$

$$x \equiv 14 \pmod{31}$$

- (b) Bob uses $N = 247 = (13)(19)$ and $r = 5$ as his encryption exponent. Find his decryption exponent. Why can't he use the smaller value 3 for r ?

For extra credit: Alice sends Bob the coded message 100. What was her original message? Do not attempt this unless finished with the exam.

8. For each of the following degree sequences, draw a graph with that degree sequence or explain briefly why there is no such graph.
- (a) 4, 2, 2, 2, 1
 - (b) 4, 2, 2, 1, 1
 - (c) 5, 4, 3, 2
 - (d) 2,2,2,1,1 (there are graphs like this: draw two different ones, one connected and one disconnected).

9. One of the two pictured graphs admits an Eulerian walk and one does not. For the graph which does have it, give the Eulerian walk as a list of vertices visited in order.

For the graph which does not have a walk, explain why it does not have an Eulerian walk.

10. One of the two pictured graphs can be colored with two colors and one cannot. Show a coloring with two colors of the one that can be colored (label each vertex as red or green) and list the vertices of an odd cycle in the other graph (in a correct order for walking through the cycle).