

Math 187 Test III

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This test will last from 7:50 am to 9:30 pm. You are allowed to use a plain scientific calculator with no graphing or symbolic computation capabilities. Your grade will be posted on the web by the ID number on the first page of your paper. Cell phones must be turned off and out of sight.

1. Do two of the three mathematical induction problems. These count as two separate problems on the test, not as two parts of one question. If you do all three, your best two problems will count, and success on all three may yield some extra credit (don't try for a third until you have completed the rest of the test!)
 - (a) Prove by mathematical induction that the sum of the first n numbers is $\frac{n(n+1)}{2}$.

- (b) Prove by mathematical induction that for each natural number n , $4^n - 1$ is divisible by 3. You may use the fact that the sum or difference of two numbers both divisible by 3 is also divisible by 3.

- (c) Prove by mathematical induction that the sequence defined by $a_0 = 3; a_1 = 7; a_{n+2} = 5a_{n+1} - 6a_n$ satisfies $a_n = 2^{n+1} + 3^n$ for each natural number n .

2. Compute the first six terms of the sequence defined by the recurrence relation

$$a_0 = 1; a_1 = 1; a_{n+2} = 3a_{n+1} - 2a_n.$$

Find a formula for this sequence.

3. Write the function defined by $f(x) = 2x + 1$ with domain $\{1, 2, 3, 4\}$ in set notation (listing all of its elements). What is the image (or range) of this function? Write the image in set notation, listing all of its elements.

4. How many functions are there from $A = \{2, 3, 4\}$ to $B = \{5, 6, 7, 8, 9\}$? How many of these functions are one-to-one? How many of them are onto B ?

Write one of these functions which is not one-to-one and explain why it is not one-to-one (mentioning specific elements of A).

Write one of these functions which is not onto B and explain why it is not onto B (mentioning specific elements of B).

5. Use the Pigeonhole Principle to explain why any collection of 12 positive integers must contain two numbers whose difference is divisible by 10. Your answer should make it clear that you understand what the Pigeonhole Principle says (you may express it informally).

6. Two functions f and g are given in set notation. Compute each of f^{-1} and g^{-1} or explain why it does not exist (be specific in your explanation, mentioning specific numbers). Compute each of $f \circ g$ and $g \circ f$ or explain why it does not exist (again, mention specific numbers in your explanation). Functions in this problem should be written in set notation, listing all their elements.

It might help to draw arrow diagrams of the functions.

$$f = \{(1, 3), (2, 2), (3, 5), (4, 4)\}$$

$$g = \{(2, 1), (3, 1), (4, 5), (5, 5)\}$$

7. One permutation σ is given in array notation and one permutation τ is given in cycle notation. Convert σ to cycle notation and τ to array notation.

$$\sigma = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 3 & 1 & 5 & 7 & 6 & 4 \end{bmatrix}$$

$$\tau = (1245)(367)$$

Each of the following computations can be done in array notation or cycle notation as you prefer: compute σ^{-1} and $\sigma \circ \tau$. Be careful about the order of composition!

(problem continues next page)

Write each of σ and τ as a composition of transpositions, and classify each as an even or odd permutation.

What is the smallest positive integer k such that ρ^k is the identity function, where ρ is the permutation $(2345)(1678910)$ in S_{10} ? Explain briefly.