Math 187 Test III (Summer 2007)

Dr. Holmes

October 28, 2008

This exam starts at 7:30 am and ends at 9:10 am. You can use (and will probably need) a scientific calculator without graphing or symbolic computation capabilities. Cell phones must be turned off and out of sight.

1. Do both of the math induction problems. The one on which you do better will count twice as much as the other. In each problem, clearly label the basis step, the induction hypothesis, and the goal of the induction step. Indicate clearly where the induction hypothesis is used in the proof of the induction step.

   (a) Prove by induction that the sum of the first n odd numbers is n^2, for any positive integer n.

(b) Prove by induction that $10^n - 1$ is divisible by 9 for each natural number n.

2. Compute the first six terms of the sequence with the recursive definition $a_0 = 1; a_1 = 2; a_{n+2} = 4a_{n+1} - 3a_n$.

   Determine an ordinary mathematical formula for the nth term of this sequence.

3. Let $A = \{a, b, c, d, e\}$ and $B = \{1, 2, 3, 4\}$.

   (a) How many functions are there from A to B?

   (b) How many functions are there from B to A?

   (c) How many one-to-one functions are there from A to B?

   (d) How many one-to-one functions are there from B to A?

   (e) Give a function from A to B written as a set of ordered pairs which is neither one-to-one nor onto.

   (f) Explain why the function you chose is not one-to-one, referring to specific value(s) in A.

   (g) Explain why the function you chose is not onto B, referring to specific values in B (not to general facts about A and B).

4. Explain using the Pigeonhole Principle why the following is true: if we choose 10 points in a 3 inch by 3 inch square, there must be two points of the 10 which are at distance less than or equal to $\sqrt{2}$ from each other.

5. Two permutations σ and τ are given, one in array notation and one in cycle notation. Compute σ^{-1}, τ^{-1}, $\sigma \circ \tau$ and $\tau \circ \sigma$ (labelling each answer clearly). You may write these permutations in whichever notation you prefer.

$$\sigma = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 5 & 6 & 4 & 3 & 1 \end{bmatrix}$$

$\tau = (1, 4, 3)(2, 6, 5)$

6. Use the Euclidean algorithm to compute $\gcd(180, 125)$; further, use the Euclidean algorithm computation to find integers x and y such that $180x + 125y = \gcd(180, 125)$. Use the table format used in class examples and show all work.

   Your answer must clearly show that you know what $\gcd(180, 125)$ is, what x is, and what y is.