Assignment X, Math 187

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March 8, 2004

This assignment is due Friday, March 12.

1. Write out a complete proof that if $c|a$ and $c|b$, then $c|xa + yb$, where x and y are any integers. You may assume familiar algebraic properties of the integers.

2. Convert $54_{\mathtt{ten}}$ and $31_{\mathtt{ten}}$ to base 6.

   Construct addition and multiplication tables for base 6.

   Add and multiply the base six versions of $54_{\mathtt{ten}}$ and $31_{\mathtt{ten}}$, carrying out all calculations in base six.

   Convert the results back to base ten and check that you have the correct answers.

3. Compute $\mathtt{gcd}(5898{,}1113)$ using the Euclidean algorithm.

   Find integers x and y such that $5898x + 1113y = 9$.

   Is it possible to find integers x and y such that $5898x + 1113y = 20$? If not, why not?

4. Let F_i be the ith Fibonacci number (as usual). Prove that $\mathtt{gcd}(F_i, F_{i+1}) = 1$ for every i. Hint: use proof by induction.

   extra credit: Prove that the gcd of any two Fibonacci numbers is a Fibonacci number.

5. Suppose we have a set of n weights, with mass 1,2,4,8,$\ldots$, 2^{n-1}. Show that it is possible to balance any object whose mass is an integral number of grams in the range from 1 to $2^n - 1$ grams, and that no other set of n weights will do this.

Proof by induction might help.

In the first part of this problem, there is an implicit assumption that the object to be weighed is to be placed on one side of the balance and that all other weights are to be placed on the other side of the balance from the object being weighed. What objects can we weigh exactly using weights 1,3,9,27 if we are allowed to place weights on either side of the balance? Does this suggest a general result like the one in the first paragraph?

6. Define a relation R on the positive integers by $x \, R \, y$ iff $\texttt{gcd}(x, y) > 1$. Is this relation symmetric? reflexive? transitive? Explain in each case why it does or does not have the property.