Math 187 Review Sheet for Final Exam

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May 6, 2004

The length of the exam will be no more than half again that of an hour exam (10 to 12 questions). You can look at the tests to get an idea of the kind of questions I might ask, as well as the comments below (this is not to say that every question will look like a question on one of the exams).

The exam will be open book, open notes (including test and homework papers if you wish).

calculator use will be permitted, but all calculations need to be set up on your paper so that I can see what you did. You need to follow instructions as to what calculations need to be done explicitly.

chapter 2: Be able to read set notation.

Know about subsets. Know that to prove $A \subseteq B$, you need to prove that an arbitrarily chosen $x \in A$ must also be in B, and to show $A = B$ (when A and B are sets) we need to show that $A \subseteq B$ and $B \subseteq A$.

Know about union, intersection, set difference. Be able to prove statements or give counterexamples using Venn diagrams.

chapter 3: Be able to do truth table reasoning about any of the logical operations.

Be able to read and write statements using the logical operations.

Understand converses and contrapositives.

Know how the proof of a statement of a certain logical form should be set up. For example, to prove $P \rightarrow Q$, assume P and use it to prove Q. (of course, one could also assume $\neg Q$ and use it to prove $\neg P$.)

Be able to read and write statements using quantifiers (for all x, for some x).

Be aware how a proof of a quantified statement should start. For example, a plan for proving $(\forall x \in calN.P(x) \rightarrow Q(x))$ might look like this:

Let x be a natural number.

Assume $P(x)$.

Goal: deduce $Q(x)$.

I might use matching questions for translations to and from logical notation.

chapter 4: If I ask for a proof using the laws of algebra, you will of course have the laws themselves handy.

Review induction proofs of statements about algebra, divisibility or sums.

Be able to read and apply recursive definitions (such as the definition of the Fibonacci numbers).

No alternative forms of induction.

chapter 5: Be able to read and draw arrow diagrams for functions (you might draw an arrow diagram of the composition of two functions given by arrow diagrams, or of a function defined by a formula on a finite set).

Know what surjections (onto maps), injections (one-to-one maps) and bijections are. I might ask for simple reasoning about them.

Know how to count injections and bijections (a very simple question about counting surjections might appear).

Know about compositions and inverses of functions, and how to represent them using arrow diagrams (you should also know the Math 147 procedures for delaing with compositions and inverses of functions given by formulas).

Be aware of the pigeonhole principle: if I put m things in n boxes and $m > n$, then more than one thing went into one box.

I might ask for a diagram of a bijection involving infinite sets.

chapter 6: Understand the definition of "the set S has m members" (there is a bijection between S and $\mathcal{N}_m = \{1, \ldots, m\}$.

How would you use bijections to define the concept of a set A and a set B having the same number of members?

How would you show that the relation of having the same number of members is an equivalence relation (hint: this has to do with properties of bijections).

chapter 7: Know the basic properties of equivalence relations (reflexive, symmetric, transitive) and what they mean.

Be able to associate a partition (collection of disjoint sets) with an equivalence relation.

I might talk about proving that a given relation is an equivalence relation (or isn't); I might also ask you to write down what needs to be proved to show that a relation is an equivalence relation (without using the words "reflexive", "symmetric", or "transitive".

Be aware of the definitions of the equivalence relation and of addition and multiplication in the construction of the integers (of course you can look them up...)

I won't ask you to verify any hard property of either of the constructions, but I might ask you to verify an easy one or to do something else that demonstrates some understanding of the constructions.

chapter 8: Be aware of the division algorithm (for any a and b, there's just one way to express a in the form $bq + r$, $0 \leq r < b$, where all numbers are integers.

Know what the greatest common divisor is, how to compute $\texttt{gcd}(a, b)$ using the Euclidean algorithm, and how to find m and n such that $\texttt{gcd}(a, b) = ma + nb$

The only property of primes that I want you to be aware of is the one that says that if $p|ab$ (p a prime) then either $p|a$ or $p|b$. Also be aware of why p must be prime for this to be true.

Calculations with other bases will probably appear. I will probably not do "decimals" in other bases, though; just conversion of integers to other bases and addition or multiplication in another base.

chapter 9: Be aware of the basic definitions for the construction of the rationals (this should be familiar). Addition, multiplication, equality (the equivalence relation) and order should all be familiar.

Be able to compute decimal representations of fractions (we'll do this only in base 10).

Be aware that the rationals are the numbers represented by terminating and repeating decimals.

Be aware of why the set of rationals is countable. Be aware of why the set of real numbers is uncountable (the Cantor diagonal construction allows us to find, given a numbered list of reals, a real not on the list).

chapter 10: Basic principles of counting: addition (add disjoint sets); multiplication (when successive choices are being made; be aware that you can draw tree diagrams). No sets of pairs or Euler's function.

Counting injections ("permutations", ordered selections without repetition). Also of course be able to handle ordered selections with repetition (powers).

Permutations: be able to convert back and forth between arrow diagrams and cycle notation. Be able to compute compositions and inverses of permutations given in cycle notation. Be able to determine how many times any "shuffle" defined by a permutation in cycle notation will need to be repeated to restore the original order.

Always be able to show how to compute binomial numbers or numbers of ordered selections using addition, subtraction, multiplication and division.

chapter 11: binomial numbers used to handle unordered choices, with or without repetition.

The sieve principle.

chapter 15: Anything we did on test 4; I'm only listing things here that we didn't do on test 4, so take a look at it for other topics.

Be able to draw a graph given an adjacency chart. Be able to judge whether two graphs are isomorphic and give an isomorphism (via an arrow diagram, say)

We didn't do anything with the concept of "planar graph"; it might show up.

I didn't ask directly about the greedy algorithm for vertex coloring; I might this time. Be sure to remember that you need to indicate the order in which you take the vertices.

chapter 16 (and 14): Something about the relationship between height and number of leaves in an m-ary tree.

The only kind of question I might ask about $O(f(n))$ notation is to ask you to make a table comparing performance of algorithms with described behaviors, showing which performance is better.

You might be asked to trace the behavior of bubble sort.

Similarly, you might be asked to trace the behavior of a heap sort; this will involve understanding how to arrange the numbers in a list into a tree, then how to "bubble" a top element to where it belongs in a heap, how to turn an unsorted list into a heap, and how to completely sort a list using heap sort. Obviously I can't do a huge example!