This document will contain some discussion of what to study for the pending Test III. It will not be as helpful as the Test II review; I’m not making up a whole slew of sample problems this time, though I may mention problems and proofs which will actually appear on the test, as before.

The sections you should study are 11, 14, 15, and 17. It is possible that some question may use the statements of theorems 18.1 and 18.2 (but not details of their proofs). I do not consider this terribly likely.

I will be available for special office hours 3-5 pm Thursday. I might be around earlier, as I will not be attending my usual 1-3 pm class. I will only be available for brief questions (re this test) in my Friday morning office hour.

**Chapter 11 stuff:** Be able to prove theorem 11.2. Certainly be ready to use this result.

Be aware of the facts stated in theorem 11.3 and theorem 11.5. You do not need to know the details of the proof of 11.3. You should be able to tell me why theorem 11.3 implies theorem 11.5, though!

Since we have now defined “closed set” in class, you should be able to do exercise 11.9.

Questions like 11.2 and 11.3 are fair game though not likely.

**Chapter 14 stuff:** Be familiar with Example 1. Be able to prove that the sum of an infinite geometric series with first term $a$ and common ratio $r < 1$ is $\frac{a}{1-r}$, using the formula for the sum of a finite geometric series and the definition of the sum of a series as a limit.

Be able to evaluate sums of actual geometric series.
Be able to use the Cauchy criterion: this will really mostly occur in the form “if \( \lim a_n \) is undefined or equal to something other than 0, then \( \Sigma a_n \) diverges.”

Be able to explain why the converse of the statement “If \( \Sigma a_n \) converges then \( \lim a_n = 0 \)” is not true.

Be able to use the Comparison Test, Ratio Test, and Root Test. You will receive no credit if you just say that a series converges by a certain test; you do need to explain the details. When computing limits in applications of the Ratio and Root tests, you are allowed to use your calculus skills.

You should be able to recognize cases where these tests do not give you any information.

The problems from the homework (14.2, 14.12, 14.13) should be understood.

In 14.13, there is a subtle point about “telescoping series”.

**Theorem:** If \( a_i \) is a sequence of real numbers with \( \lim a_n = 0 \) then

\[
\Sigma_{k=1}^{\infty} (a_k - a_{k+1}) = a_1
\]

**Proof:** Notice that

\[
\Sigma_{k=1}^{n} (a_k - a_{k+1}) = a_1 - a_{n+1}
\]

, since all intervening \( a_i \)’s cancel in \( (a_1 - a_2) + (a_2 - a_3) + \ldots + (a_{n-1} - a_n) + (a_n - a_{n+1}) \).

Now \( \Sigma_{k=1}^{\infty} (a_k - a_{k+1}) = \lim_{n \to \infty} [\Sigma_{k=1}^{n} (a_k - a_{k+1})] = \lim_{n \to \infty} [a_1 - a_{n+1}] = a_1 \), the last equation relying on the fact that we know that \( \lim a_{n+1} = \lim a_n = 0 \).

The important thing to notice is that it is not just the cancellations that do the work here: *you must note that the terms in the telescoping differences actually go to zero*. Some of you seemed to already know this, but some just wrote the infinite cancellations.

If just infinite cancellations did the trick we could easily show that \( \Sigma (-1)^n = 1 \) and also that \( \Sigma (-1)^n = 0 \) (try it!)
**chapter 15 stuff:** Be able to apply the Alternating Series Test. Understand that to conclude that $\sum(-1)^{n}a_{n}$ requires that $a_{n} \geq 0$, $(a_{n})$ is a nonincreasing sequence, and $\lim a_{n} = 0$.

I may ask you to produce an example (I gave one in class) where $a_{n} \geq 0$, $\lim a_{n} = 0$, but $\sum(-1)^{n}a_{n}$ does not converge.

You should read the proof of the Alternating Series Theorem and make sure you understand it. It is not impossible that I might ask about bits of it (I might ask about relationships between partial sums of alternating series and terms of those alternating series.)

Be able to apply the Integral Test. I will not ask you to do complicated integrals, but you should be able to apply the test to determine whether $\sum \frac{1}{n^{p}}$ converges or does not converge for any $p > 0$. Be sure that you can state all the conditions under which the Integral Test applies (the function involved must be positive, nonincreasing and the appropriate improper integral must converge/diverge).

Make sure that you understand problem 15.6, all parts (including the proof in part b).

**chapter 17 stuff:** Understand the definition of continuity and Theorem 17.2. Be able to do the homework exercises 17.3 (use theorems 17.4 and 17.5), 17.9 (in particular, be able to show continuity of $x^{2}$ at other values of $x_{0}$, not just 2), and 17.10 (re 17.10 it is an important skill to be able to spell out what it means for a function not to be continuous at a point, and I may ask you directly to write this out).

Be able to prove the theorem of homework problem 17.5 using math induction and theorem 17.4. The basis step requires you to prove that $f(x) = x$ is continuous.

Be able to prove the theorem of homework problem 17.11. I like this: it requires you to use theorem 11.3 without in any way involving details of its proof. Remember that this is a biconditional: you need to argue in both directions.

**chapter 18?** If I ask questions involving chapter 18, they will be optional and will involve applying the familiar theorems 18.1 and 18.2 rather than following the details of their proofs. Be aware that you do need to know the details of the proofs of theorems 18.1 and 18.2 for the final exam, though.