This test has pages 1–2. Take a moment to make sure you have them all.

1  Ten points each:

(a) State the formal definition of \( \lim(s_n) = L \) for sequence \((s_n)_{n=1}^{\infty}\). The best answer here uses logical symbols such as \( \exists, \forall, \text{ and } \implies \).

(b) Write down the formal statement of the Archimedean Property of Order in \( \mathbb{R} \). Again the best answer uses logical symbols such as \( \exists, \forall, \text{ and } \implies \).

(c) Write down the definition of the sentence

\[ z \text{ is an upper bound of set } A. \]

The best answer also uses logical symbols.

(d) Write down the formal negation, or denial, of your part-(c) answer. This means that you will be writing down what is true if it is not the case that \( z \) is an upper bound of \( A \).

2  Write up a formal mathematical-induction proof of the assertion

\[ \sum_{k=1}^{n} (2k - 1) = n^2. \]

Write so as not to attract the attention of the Circular-Reasoning Police. Your proof must be a formal mathematical-induction proof. (20 points)

3  Let the sequence \((a_n)_{n=1}^{\infty}\) be given by

\[ a_n = \frac{30n + 5}{6n - 29}. \]

Write a proof of the value of \( \lim(a_n) \). Your proof must work directly from the definition of \( \lim(a_n) \). (20 points)
4 Pick one of the following assertions (a)-(d) (say which) and write up a formal proof of the chosen assertion.

(a) Let \((a_n)^\infty_{n=1}\) and \((b_n)^\infty_{n=1}\) be convergent sequences. Prove that \((c_n)^\infty_{n=1}\) is also a convergent sequence, where \(c_n = a_n + b_n\). (20 points)

(b) Let \(S\) and \(T\) be nonempty bounded subsets of \(\mathbb{R}\). Suppose, moreover, that \(s \leq t\) whenever \(s \in S\) and \(t \in T\). Prove that
\[
\sup(S) \leq \inf(T).
\]
(20 points)

(c) Let \((a_n)^\infty_{n=1}\) be a sequence and let \(\lim(a_n) = L\), where \(L \in \mathbb{R}\).
Suppose also that \(M \in \mathbb{R}\) and that, for all \(n\), \(a_n < M\).
Prove that \(L \leq M\). (25 points)

(d) Let \((a_n)^\infty_{n=1}\), \((b_n)^\infty_{n=1}\), and \((z_n)^\infty_{n=1}\) be sequences.
Suppose that \((a_n)\) and \((b_n)\) are convergent sequences with identical limits.
Suppose also that there exists a natural number \(K\) such that \(n > K\) implies
\[
a_n \leq z_n \leq b_n.
\]
Prove that \((z_n)\) is also a convergent sequence. (25 points)