Math 333 Test One

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The test will begin at 2:40 pm and end at 3:35 pm.
You may use a plain scientific calculator without graphing or symbolic computation capabilities; I don’t believe that there is any problem on the exam (even the word problem) which actually requires any calculator use. You may not use any other calculator, and you may not use a cell phone or PDA (such as a Palm Pilot) as a calculator (this is stated in the course syllabus).
Books, notes, and neighbors are to remain firmly closed.
Show all work.
1. Sketch the direction field for the differential equation $y' = 1 - y^2 = (1 - y)(1 + y)$ and sketch in some solution curves. Your solution curves should demonstrate each of the qualitatively different kinds of solution.
2. Show calculations verifying the fact that $y = \tan^2(x)$ is a solution of the differential equation

$$(y')^2 = 4y(1 + y)^2.$$ 

Please notice that you are not being asked to solve this equation!

You will need to remember the trigonometric identity relating secant to tangent.
3. Find the general solution for the differential equation

\[(x^2 + 1)y' = y - 2.\]

Is there a singular solution (and if not, why not?). Solve the initial value problem

\[(x^2 + 1)y' = y - 2;\]

\[y(0) = 4\]
4. Solve the linear differential equation

\[ ty' = y + t \]

Hint: you need to put this into the right algebraic form before you can apply the technique of solution!
5. A tank with a capacity of 80 gallons initially contains 40 gallons of salt water containing 0.2 lb of salt per gallon. Pure water is added to the tank at eight gallons per minute; at the same time, solution drains out of the tank at four gallons per minute.

Set up the initial value problem which we would need to solve to determine the function \( x(t) \) defined as the amount of salt in the tank at time \( t \). Set up the computation of the concentration of salt (in pounds per gallon) in the tank at the moment when it fills, using this function.

Complete the calculations and determine the concentration of salt in the tank at the moment when it fills.
6. Solve the initial value problem

\[ xy' = y^2 - 9; \]

\[ y(1) = 3 \]