

Math 333 Test IV and Final, Summer 2009

Dr. Holmes

July 29, 2009

This exam will begin at 11:50 am and end at 1:50 pm. You are allowed your book, one standard sized sheet of paper with whatever you like written on it, and your non-graphing calculator.

The usual remarks about blue books and magic numbers apply.

If you have trouble due to the length of the exam, remember that the adjustment I am likely to make (depending on class performance) is to drop or reduce the value of the lowest question on each part. I'm not saying I *will* do that, just that if I make an adjustment that is what it will look like.

1 Test IV

1. Tank A has a capacity of 100 gallons and initially contains 100 gallons of solution containing 3 lbs of salt. Tank B has a capacity of 50 gallons and initially contains 50 gallons of pure water. A pipe carries solution from tank A to tank B at 2 gal/sec and another pipe carries solution back from tank B to tank A at 2 gal/sec. Write a system of equations (with initial values) describing this situation. Write it both as a system of two equations in two unknowns with initial values and as a single initial value problem in matrix form.

If you can solve this, you will get substantial extra credit for doing so. I would describe it as doable, though with annoying fractions. Don't attempt this unless you are done with the rest of the exam.

2. Solve the system

$$x_1' = x_1 - 2x_2$$

$$x_2' = x_1 + 4x_2$$

State the general solution. Your final answer should not involve matrix or vector notation.

Find the particular solution which satisfies $x_1(0) = 1; x_2(0) = -1$.

This is the real eigenvalue case.

3. Solve the system

$$x_1' = x_1 - 4x_2$$

$$x_2' = x_1 + x_2$$

State the general solution. Your final answer should not involve matrix or vector notation.

This is the complex eigenvalue case, and your final answers should be in real form.

4. Do the matching problem handwritten on the next page.

2 Cumulative Part

1. Solve the separable differential equation

$$y' = y^4.$$

Show all work and find *all* solutions.

2. Solve the linear inhomogeneous initial value problem

$$y' = 3y + t :$$

give the general solution.

3. An electric circuit has a coil with inductance 1, a resistor with resistance 5, and a capacitor with capacitance $\frac{1}{4}$. An electromotive force of $\cos(2t)$ volts is supplied. The charge on the capacitor is initially 2, and the current is initially zero. Find an expression for the current in the circuit at time t . You may use any method you wish to solve the equation.
4. Solve the following initial value problem by the method of Laplace transforms:

$$y'' + 3y' + 2y = e^{2t}; y(0) = 0; y'(0) = 1$$

5. If y is the solution to

$$y' = y + t; y(0) = 3,$$

estimate $y(1)$ using two steps of the second-order Runge-Kutta method.