

Math 333 Test III, Summer '09

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This exam begins at 11:50 am and ends at 1:30 pm. You may use a standard scientific calculator without graphing or symbolic computation (in fact, you need one). Cell phones must be turned off and out of sight.

The usual statements about Test ID numbers and blue books should be understood by now.

1. Solve $y'' + 3y' + 2y = e^t$ by variation of parameters (use of any other method carries little if any credit). Find a particular solution using the method of variation of parameters, then write down the general solution.

The following equations are provided as a reminder. You should know what everything means and how to use it.

$$\begin{aligned}v_1' y_1 + v_2' y_2 &= 0 \\v_1' y_1' + v_2' y_2' &= g(t)\end{aligned}$$

2. Compute the Laplace transform of e^{3t} using the definition.
3. Compute the inverse Laplace transforms of the following functions of s .

(a)

$$\frac{1}{s(s+3)}$$

(b)

$$\frac{s+3}{s(s^2+1)}$$

(c)

$$\frac{e^{-2s}}{s^2+4}$$

4. Solve the following initial value problem using the method of Laplace transforms:

$$y'' + 3y' + 2y = e^t; y(0) = -1; y'(0) = 1$$

Solving it by any other method carries very little credit.

5. Solve the following initial value problem using the method of Laplace transforms:

$$y' - y = f(t); y(0) = y'(0) = 0,$$

where

$$f(t) = \begin{cases} 3 & t \leq 2 \\ 0 & t > 2 \end{cases}$$

Write your final answer as a piecewise defined function without using Heaviside functions.

You may if you wish solve this problem using the methods of the last test; this will carry substantial credit, though not full credit.

6. If y is the solution of the initial value problem

$$y' = x + y; y(0) = 1$$

estimate $y(2)$ using Euler's method with 4 steps. Use the maximum accuracy your calculator supports in all calculations.

7. An electrical circuit has a coil with inductance 1, a resistor with resistance 4, and a capacitor with capacitance $\frac{1}{3}$. At $t = 0$, both the current and the charge on the capacitor are 0. An electromotive force equal to $\cos(t)$ is supplied.

Find expressions for the charge on the capacitor and for the current at time t .

A basic equation supplied as a reminder:

$$LQ'' + RQ' + \frac{1}{C}Q = E(t)$$

Remember that current is the derivative of charge.

Identify the steady-state component of the solution (this continues to be significant for large values of t) and the transient component (which becomes negligible for large values of t).

You may use any method we have learned to solve this problem.

Some Laplace transforms

$$L(1) = \frac{1}{s}$$

$$L(t) = \frac{1}{s^2}$$

$$L(t^n) = \frac{n!}{s^{n+1}}$$

$$L(e^{at}) = \frac{1}{s - a}$$

$$L(\sin(at)) = \frac{a}{s^2 + a^2}$$

$$L(\cos(at)) = \frac{s}{s^2 + a^2}$$

$$L(f(t - a)H(t - a)) = e^{-cs}L(f(t))$$

[as I have remarked, this is not always the optimal formula to use].