Math 333 Test 3

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You will have two days to work on this test. You may not use a calculator with graphing or symbolic computation capabilities; you are likely to need a plain scientific calculator. Books, notes, and neighbors to remain closed. Good luck!

1. Solve the differential equation

$$y'' + y' - 2y = e^t$$

using the method of variation of parameters. Use of any other method carries very little credit.

The promised equations are provided (it's your responsibility to understand what they mean):

$$\begin{aligned} v_1' y_1 + v_2' y_2 &= 0 \\ v_1' y_1' + v_2' y_2' &= g(t) \end{aligned}$$

2. An electric circuit containing a coil with inductance 1, a resistor with resistance 5 and a capacitor with capacitance $\frac{1}{6}$ is supplied with electromotive force $\sin(t)$. The current and the charge on the capacitor at $t = 0$ are both 0.

   Determine the charge on the capacitor as a function of t. Determine the current in the circuit as a function of t.

   Identify the steady-state part and the transient part of the solution for the current.

3. Laplace transforms

(a) Compute the Laplace transform of e^{-t} using the definition

$$L\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt$$

of the Laplace transform (show all work).

(b) Compute the Laplace transforms of the following functions. Show any work that you need to do (here you are working from tables and rules, not from the definition!).

i.
$$2\cos(3t)$$

ii.
$$t^2 e^{-t}$$

iii.
$$t\sin(t)$$

4. Let y be the solution of the initial value problem

$$y' = x - y; y(1) = 2;$$

estimate $y(2)$ using two steps of the Runge-Kutta method.

5. Use the method of Laplace transforms to solve the initial value problem. Use of any other method carries very little credit.

$$y'' + 4y' + 3y = e^{2t}; y(0) = 0; y'(0) = 1$$

6. Use the method of Laplace transforms to solve the initial value problem

$$y' + 4y = f(t); y(0) = 0; y'(0) = 0$$

where $f(t) = 2$ for $t < 3$ and $f(t) = 0$ for $t \geq 3$. Write your final answer as a piecewise defined function (without use of Heaviside functions).

7. Compute the inverse Laplace transforms of the following functions. Show any work you need to do.

(a)

$$\frac{s}{(s+1)(s^2+1)}$$

(b)

$$\frac{e^{-2s}}{s-2}$$

Write your answer as a piecewise defined function (without use of Heaviside functions).

(c)

$$\frac{1}{s^2+4s+5}$$

8. Solution curves of a differential equation are pictured. Suppose we are given the value of a solution at 0 and we estimate the value of the solution at 2 using several steps of Euler's method. Will our estimate be an overestimate or an underestimate? Explain your answer by drawing a picture of what happens when Euler's method is applied.