Math 333 Test Two

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You will have from 2:40 pm to 3:35 pm to take this exam. You may use a standard scientific calculator without graphing or symbolic computation capabilities. You may not use any other calculator, nor may you use a cell phone or PDA as a calculator. **Turn off your cell phone now.** Books, notes and neighbors to remain firmly closed.
1. If you start a bank account now, starting with a balance of zero and paying money into it continuously at a rate of $1,000 per year, and the account earns 8% annually, compounded continuously, how much money will you have in 20 years?
2. Two equations are given. One is an exact differential equation (after some fiddling to get it into differential form) and the other is not. Explain why the one which is not exact is not exact and solve the one which is exact. You may leave your answer in implicit form.

(a) 
\[(3x^2 + 4xy) + (2x^2 + 1)y' = 0\]

(b) 
\[(3x^2 - 4xy) + (2x^2 - 1)y' = 0\]
3. Solve the differential equation

\[ y'' + 4y' + 3y = 0. \]

Give the general solution.

(for just a point or two) write two different nonzero solutions to this equation which are not linearly independent. You don’t need to do anything with Wronskians!
4. Find a particular solution to the differential equation

\[ x'' + 4x' + 4x = \cos(t) \]

then write out the general solution.
5. Suppose that you set up an electric circuit with a 1 henry coil, a 2 ohm resistor, and a 0.5 farad capacitor. The capacitor is initially charged to 10 volts and there is no current initially in the circuit. Write an expression for the current in the circuit as a function of time (but be aware that initially you will be solving for the charge on the capacitor as a function of time!)
6. Do one of the two following problems. If you do both your best work will be marked.

(a) Verify that \( y = 0 \) and \( y = (\frac{2}{3}t)^{\frac{3}{2}} \) are both solutions of the initial value problem
\[
y' = y^{\frac{1}{3}}, \; x(0) = 0.
\]
This means that you should check that the given functions are solutions of the initial value problem, not that you should go through solving it.
Explain why this does not contradict the uniqueness theorem; your answer should make it clear that you know the statement of the uniqueness theorem.
(b) Write the differential equation

\[ y'' + 2y' - 3y = 0 \]

as a system of two first-order equations. Sketch part of its direction field, by sketching vectors at the nine points with \( y \) and \( v \) coordinates of 0 or ±1. Supply supporting work for each vector drawn. Draw your vectors with a small scale so they won’t run into each other.