Math 333 Final Exam

Dr. Holmes

December 8, 2010

The exam begins at 3:30 pm and ends at 5:30 pm (officially) with an extension to 5:45 pm *if everyone agrees*.

This is an open book, open notes, closed neighbor exam. You are allowed to use any calculator (cell phones and palmtop computers may not be used as calculators, and **cell phones must be turned off**). Please be sure that you set up any calculator work on your page, and read instructions carefully as I will sometimes require hand calculations.

Please make a note of the number on your exam paper. This number will be used to post your final exam and course grades on my web site. I hope to post your grades by the end of the day on Tuesday.

Good luck, and have a nice holiday season.

1. Solve the equation
$$(x^2+1)y' = y^2$$
by separation of variables. Be sure to find *all* solutions, and show a complete derivation of your result by hand. This is essential for *any* credit, as my calculator can solve this equation (though it does not find all solutions).

2. Solve the linear equation and initial value problem. Show a complete derivation of your result by hand. This is essential for *any* credit, as my calculator can solve this problem.

$$y' = y + 2xe^{2x}; y(0) = 3$$

3. An electric circuit contains a capacitor with capacitance .25 units, a resistor with resistance 5 units, and a coil with inductance 1 unit. An electromotive force of $5\sin(2t)$ is supplied. The charge on the capacitor is initially 1 unit, and the current in the circuit is initially zero.

   Set up the second order linear equation and initial value problem modelling this situation and solve it by the method of undetermined coefficients. Show a complete hand derivation of your results.

   Give expressions for the charge on the capacitor and the current in the circuit at any time t.

4. You have a choice of one of the two Laplace transform problems. If you do both parts successfully, extra credit is possible.

   (a) Solve the initial value problem

$$y' - 4y = f(t); y(0) = 0$$

   where $f(t) = 5$ for $t < 2$ and $f(t) = 0$ for $t > 2$, using the method of Laplace transforms and showing all work.

(b) Solve the system of equations with the given initial conditions, using the method of Laplace transforms:

$$x' = x + 3y$$
$$y' = x - y$$

$$x(0) = 1; y(0) = 2$$

5. Estimate $y(2)$, where $y' = t^2 + y^2, y(0) = 1$, using four steps of Euler's method. Show all calculations on your paper in a table format.

   Show your final answer in the form $y(2) \sim \ldots$

6. Solve the system of differential equations using the method of eigenvalues and eigenvectors. Your work must be supported by hand calculations. Show the general solution in the form

$$x(t) = \dots$$

$$y(t) = \dots$$

i.e., without using vector notation.

The system to be solved is

$$x' = 6x + 4y$$
$$y' = -4x - 2y$$

7. Match the following systems with the pictures of direction fields on the next pages (I suggest writing the letter of the matching equation on the matching picture). Show calculations which support your answers, and classify the equilibrium point for each equation, including the one that doesn't match (i.e., write something like "nodal source" or "spiral sink"). Also classify the equilibrium point in the unmatched picture.

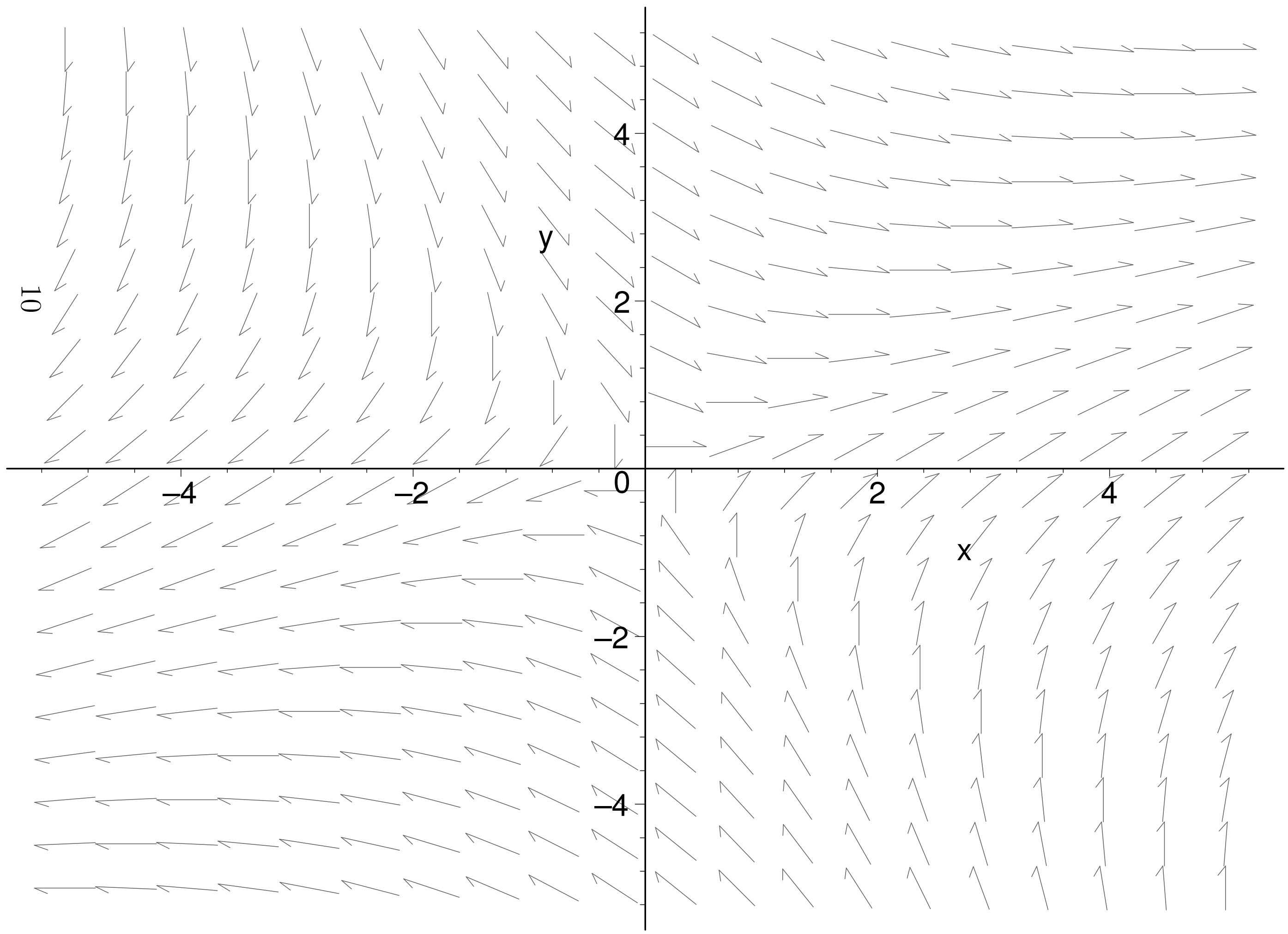
   One of the four equations does not match any picture, and of course one of the four pictures does not match any of the equations. As noted above, you need to classify the equilibrium points for the unmatched equation and picture.

   (a)
   $$x' = x + y$$
   $$x' = x - y$$

   (b)
   $$x' = 2x - 3y$$
   $$y' = 3x + 2y$$

   (c)
   $$x' = -1x + 5y$$
   $$y' = -2x + 1y$$

   (d)
   $$x' = x + 2y$$
   $$y' = -x + 4y$$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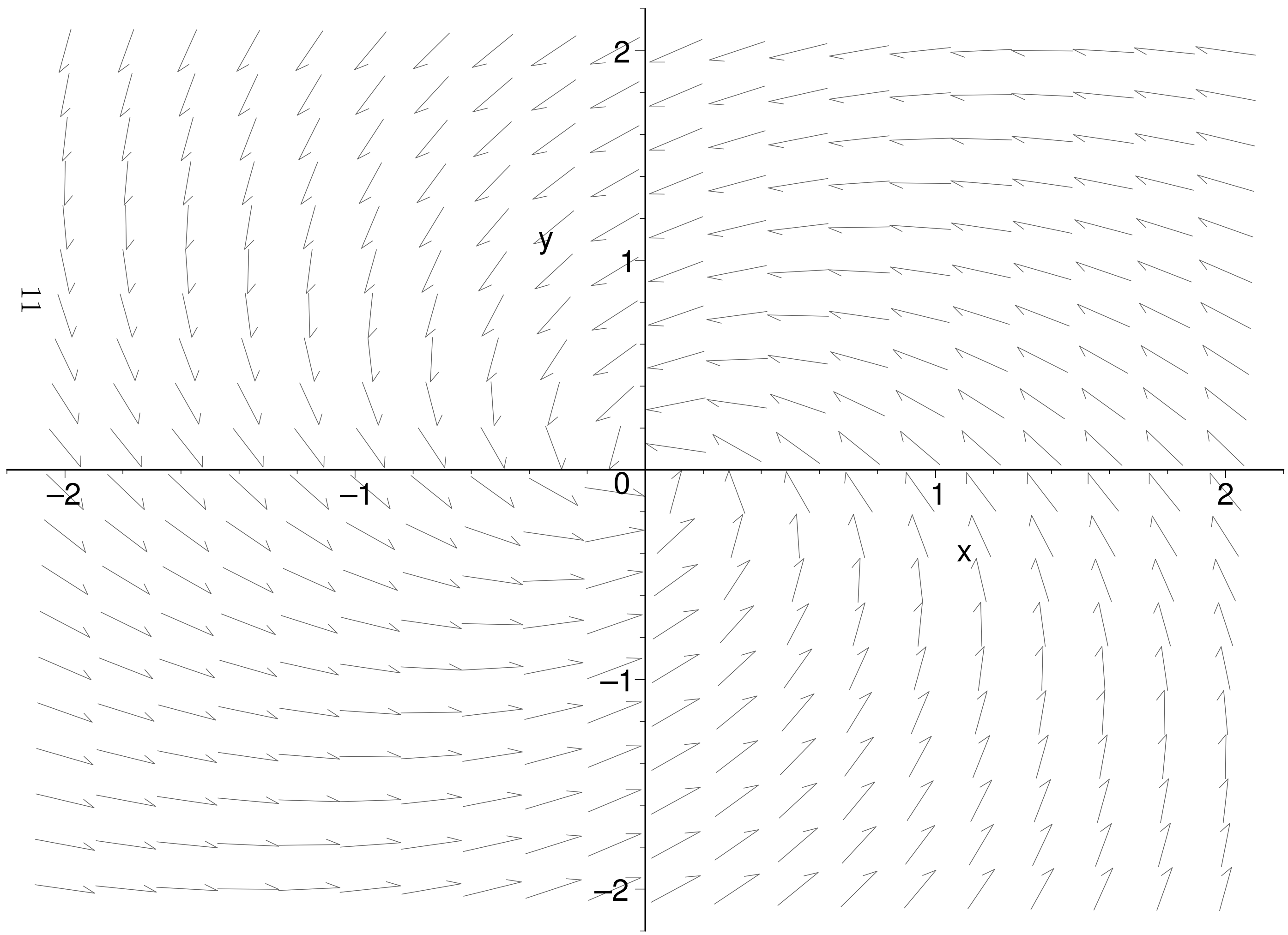
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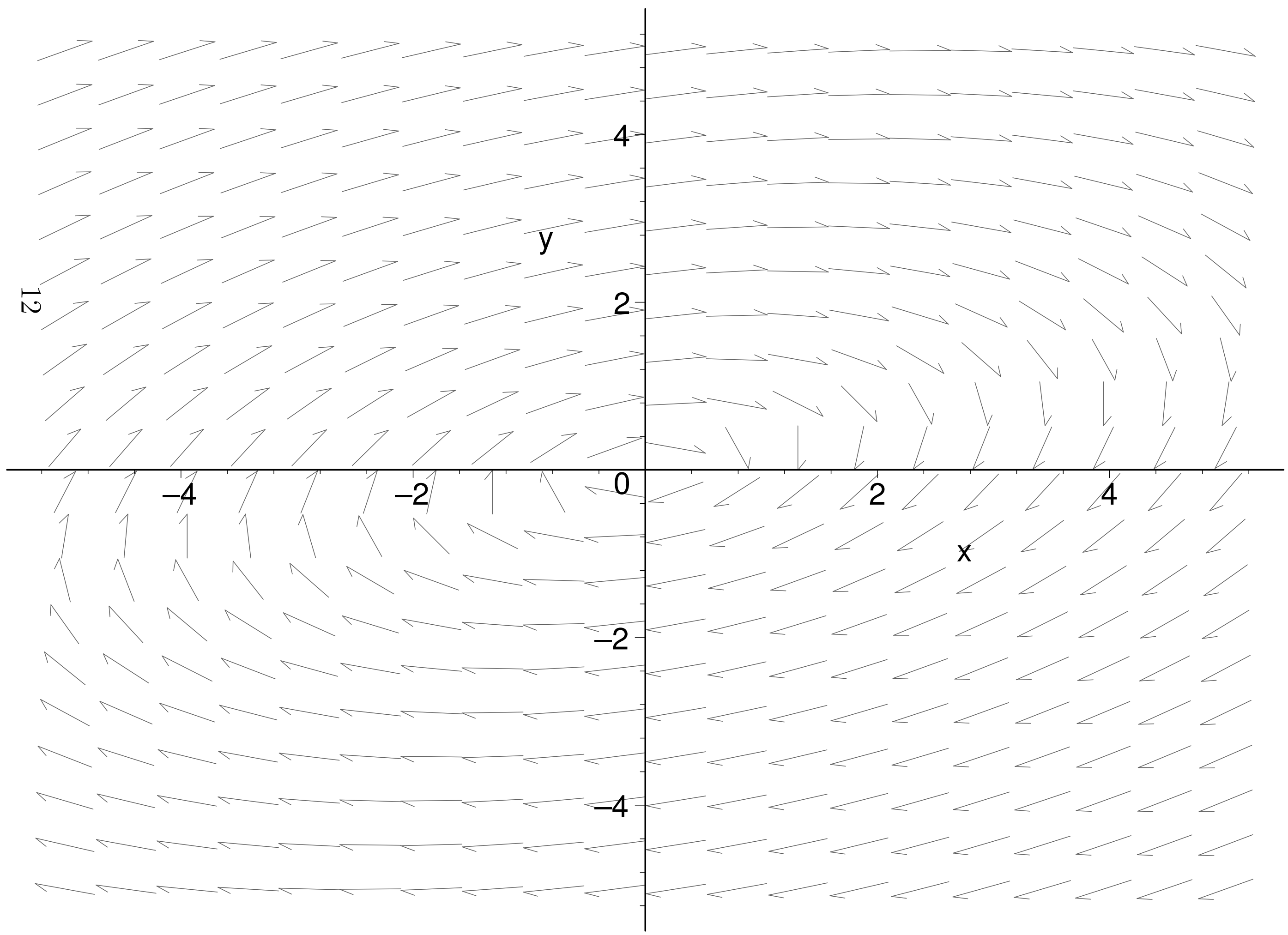
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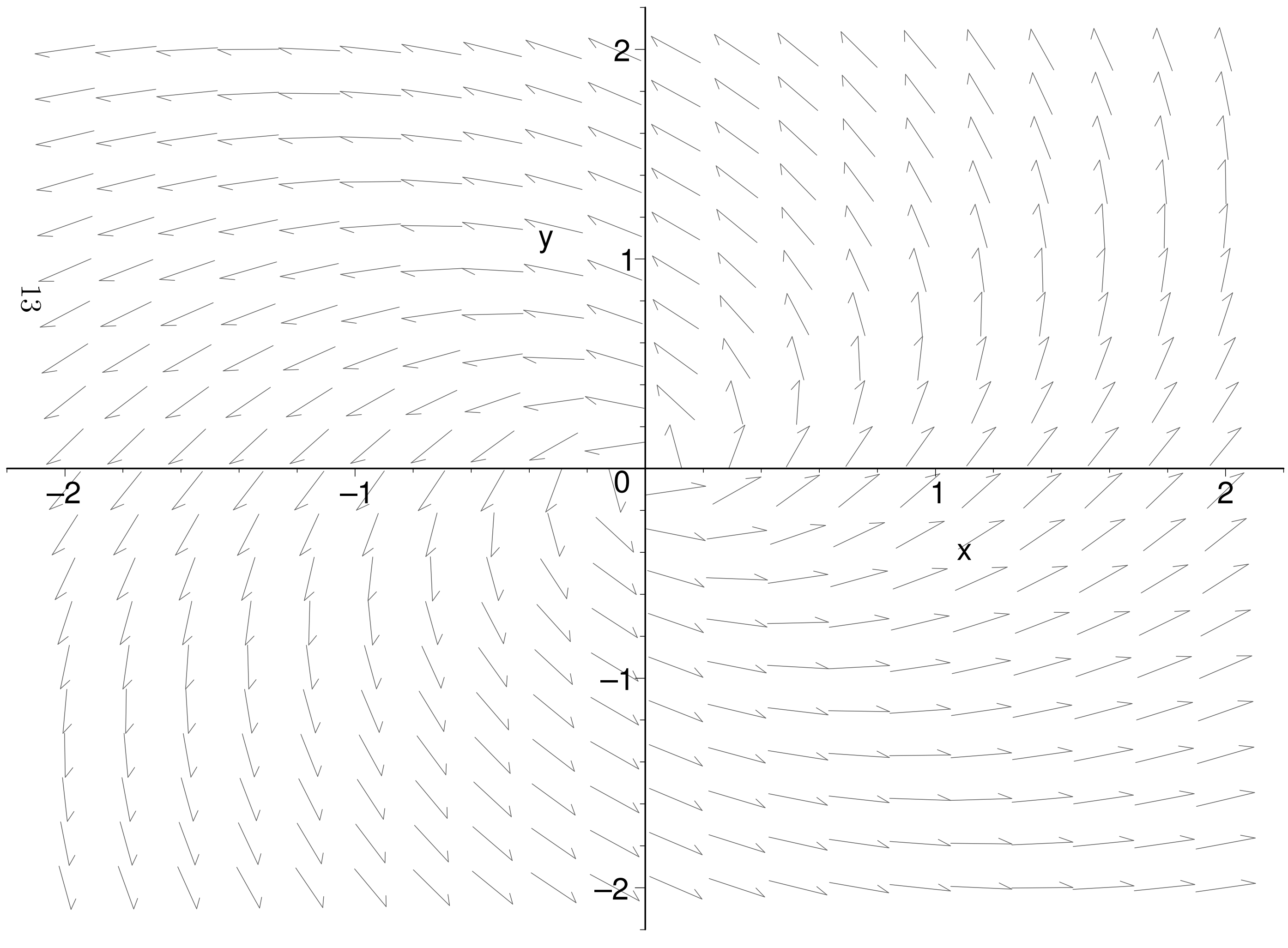
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8. Determine the general solution of the following system of equations. You may use either the method of variation of parameters or the method of undetermined coefficients. You may use your calculator, but the setup for any calculator work must be shown on your paper.

   Your final answer should not be in vector notation.

   The system of equations to be solved is

$$x' = x - 3y + e^t$$
$$y' = 2x - 4y - 1$$

   The general solution to the homogeneous system in vector form is

$$c_1 e^{-t}(3 \quad 2)^T + c_2 e^{-2t}(1 \quad 1)^T$$