1. Suppose that a second-order linear homogeneous differential equation has solutions $t+1$ and t^2. Find a solution $x(t)$ of the same equation which satisfies the initial conditions $x(1) = 5; x'(1) = 1$.

   Not a test question: Find a second-order linear homogeneous differential equation whose solutions are $t+1$ and t^2. Find the largest intervals on which solutions to this equation exist.

2. Suppose that a second-order homogeneous linear differential equation has solutions $t+1$ and t^2 on an open interval I. Use the linear independence properties of solutions of second-order homogeneous differential equations to explain why neither of the numbers 0 or -2 can be in the interval I. (If you found the equation, you will be able to see that this is true; but it is possible to explain why this has to be true without finding the equation at all!)

3. Solve the differential equation

$$x'' = \frac{3}{t}x'$$

4. I claim that one of the solutions of the differential equation

$$x'' = \frac{3}{2t}x' - \frac{1}{t^2}x$$

is t^2. Check that this is actually true. Then find another linearly independent solution by the method of reduction of order and write the general solution of the equation.

5. Solve the differential equations and initial value problems.

(a)

$$x'' + 9x = 0; x(0) = 2; x'(0) = 3$$

(b)

$$x'' = 3x' - 2x; x(0) = 3; x'(0) = 2$$

(c)

$$x'' + 25x = 10x'$$

(d) Write a second-order linear homogeneous differential equation with constant coefficients whose general solution is $c_1 e^{2t}\cos(t) + c_2 e^{2t}\sin(t)$. Hint: what would the roots of the characteristic polynomial need to be? Use the roots to compute the characteristic polynomial itself (and so to find the original equation).

6. Solve the differential equation and initial value problem using the method of undetermined coefficents.

$$x'' - 4x = \cos(2t); x(0) = 1; x'(0) = -1$$

7. The system of equations to be solved in the method of variation of parameters when solving the equation

$$x'' = p(t)x' + q(t)x + r(t)$$

is

$$s_1'(t)x_1(t) + s_2'(t)x_2(t) = 0$$

$$s_1'(t)x_1'(t) + s_2'(t)x_2'(t) = r(t)$$

You will be shown this system of equations on the test; it is your responsibility to know what it means (what x_1 and x_2 are, what s_1 and s_2 are, what $r(t)$ is, and how to use this system of equations to solve the differential equation).

Use the method of variation of parameters to find a particular solution (and of course the general solution) of the equation

$$x'' = \frac{2}{t}x' - \frac{2}{t^2}x + t,$$

given the information that $x_1(t) = t$ and $x_2(t) = t^2$ are a pair of linearly independent solutions to the homogeneous equation

$$x'' = \frac{2}{t}x' - \frac{2}{t^2}x.$$

I have worked this out: it is easy if you keep your wits about you.

8. The differential equation describing a harmonic oscillator (a spring-mass system) with damping and a driving force $f(t)$ is

$$x'' = -\frac{k}{m}x - \frac{\nu}{m}x' + f(t)$$

(a) Let the spring constant be 4, the mass be 1, and assume no friction and no driving force. Set up and solve the initial value problem describing what happens if I stretch the spring 3 units and let go. Give a qualitative description of what happens in physical terms.

(b) With the same spring constant, mass and absence of driving force, determine the smallest value of the coefficient of friction ν at which the spring will fail to oscillate if I stretch it 3 units and let go.

(c) Restore a frictionless environment and apply a driving force of $\cos(2t)$. Set up and solve the initial value problem describing what happens if I stretch the spring 3 units and let it go. Give a qualitative description of what happens in physical terms.