Math 333 Test 2

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March 6, 2002

This test will begin at 7:40 am and end at 9:30 am. You are not allowed to use calculators with graphing or symbolic computation capability; you are permitted to use a standard scientific calculator. Books, notes and neighbors are to remain firmly closed.

The lowest question will be dropped. If you do all questions, your best work will count, and you may get extra credit if you are uniformly successful on all problems.

1. Do two of the three parts; your best work will count if you do all three. Suppose (in all parts) that a second-order linear homogeneous differential equation

$$x'' = p(t)x' + q(t)x$$

has solutions t^2 and t^3.

(a) Verify that these two solutions are linearly independent. If I is the open interval over which our solution is defined, explain why 0 cannot be in I.

(b) Find a solution $x(t)$ of the equation which satisfies the initial conditions $x(1) = 1; x'(1) = 0$.

(c) Find the equation.

2. One of the solutions of the differential equation $x'' - 2x' + x = 0$ is e^t. (You should know this; you should also know what the other solution(s) are, since this is an equation with constant coefficients, but we are going to solve it a different way here.)

   Use the method of reduction of order to find the general solution of this equation (i.e., make the substitution $x = ue^t$, simplify the resulting equation to a first order equation for u and solve).

3. Do two of the three parts. If you do all three the best two will count and you might get extra credit.

   (a) Solve the differential equation

   $$x'' + 16x = 0$$

   (b) Solve the initial value problem

   $$x'' = 3x' - 2x; x(0) = 1; x'(0) = 2$$

   (c) Write a second-order linear homogeneous differential equation with constant coefficients whose general solution is $c_1 e^t \cos(t) + c_2 e^t \sin(t)$. Hint: what would the roots of the characteristic polynomial need to be? Use the roots to compute the characteristic polynomial itself (and so to find the original equation).

4. The system of equations to be solved in the method of variation of parameters when solving the equation

$$x'' = p(t)x' + q(t)x + r(t)$$

is

$$s_1'(t)x_1(t) + s_2'(t)x_2(t) = 0$$

$$s_1'(t)x_1'(t) + s_2'(t)x_2'(t) = r(t)$$

It is your responsibility to know what these equations mean (what x_1 and x_2 are, what s_1 and s_2 are, what $r(t)$ is, and how to use this system of equations to solve the differential equation).

Use the method of variation of parameters to find a particular solution (and of course the general solution) of the equation

$$x'' = -\frac{1}{t}x' + \frac{1}{t^2}x + t,$$

given the information that $x_1(t) = t$ and $x_2(t) = \frac{1}{t}$ are a pair of linearly independent solutions to the homogeneous equation

$$x'' = -\frac{1}{t}x' + \frac{1}{t^2}x.$$

5. The differential equation describing a harmonic oscillator (a spring-mass system) without damping and with a driving force $f(t)$ is

$$x'' = -\frac{k}{m}x + f(t)$$

(a) Let the spring constant be 9, the mass be 1, and assume no friction and no driving force. Set up and solve the differential equation. Give a qualitative description of what happens in physical terms.

(b) Continue to assume a frictionless environment and apply a driving force of $\sin(3t)$. Set up and solve the initial value problem describing what happens if I stretch the spring 3 units and let it go. Give a qualitative description of what happens in physical terms.

6. Match the pictured phase portraits (on this and the following page) with the given second-order linear homogeneous equations with constant coefficients.

   Note that one of the "portraits" cannot be the phase portrait of any such equation. Explain briefly why it cannot.

   For each portrait of an equation, classify its equilibrium solution (as, for example, source, sink, saddle, center, spiral source, spiral sink, etc.)

   (a)
   $$x'' = -x$$

   (b)
   $$x'' = x' - x$$

   (c)
   $$x'' = x$$

   (d) not a phase portrait

7. Estimate $x(3)$, where x is the solution of the initial value problem

$$x'' = x' - x + t; x(2) = 1; x'(2) = 2$$

using two steps of Euler's method (i.e., $\Delta t = \frac{1}{2}$).