Math 333 Test 4

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The exam begins at 2:40 pm and ends at 3:35 pm on each of the two days that it is given.

You may have your test paper, your calculator, a writing instrument, and drafting tools (if desired) on your desk. Nothing else is permitted.

Use of any calculator is permitted on this exam, but be sure to provide work when it is requested. In general, if you use your calculator show the setup for your calculations on your paper.

You are not permitted to use cell phones or palmtop computers as calculators, for security reasons.

1. Compute the inverse of the matrix

$$\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$$

using row operations and showing all steps. You must describe the row operations and show the matrices on your paper.

Set up and carry out a matrix calculation using this inverse matrix to solve the system of equations

$$x + 2y = 5$$
$$2x + 3y = 9$$

2. The matrices

$$A = \begin{pmatrix} 2 & 3 \\ 0 & -1 \\ 1 & 1 \end{pmatrix}$$

and

$$B = \begin{pmatrix} 1 & -1 & 3 \\ 2 & 1 & 0 \end{pmatrix}$$

can be multiplied in either order. Compute the matrix products AB and BA (labelling each one clearly).

3. A system of tanks with various pipes transferring water with different percentages of NaCl (common salt) at different rates is shown. Set up a system of differential equations and initial value problem whose solution will describe the amount of salt in each tank at time t.

   It is good for extra credit if you can solve the initial value problem and state the amount of salt in each tank at any time t.

4. Solve the system of linear differential equations with constant coefficients using the method of eigenvectors and eigenvalues. Show hand calculations for the eigenvalues and eigenvectors.

   If the eigenvalues are complex, you need to give real solutions to the equations.

$$x' = 4x + 2y$$
$$y' = -x + y$$

   Give the general solution to this system of equations in the form

$$x(t) = \dots$$
$$y(t) = \dots$$

   i.e., the final answer should not involve any vector notation.

   Find a particular solution that satisfies the initial conditions $x(0) = 1; y(0) = 2$.

5. The set of vectors

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$$

is linearly dependent.

Verify that it is linearly dependent by computing a determinant (showing hand calculations).

Find a nontrivial linear combination of these vectors which is equal to the zero vector, using appropriate matrix methods.

6. Find and sketch the x- and y- nullclines of the system of equations and find the equilibrium solutions.

$$x' = x^2 - y^2$$
$$y' = (x+1)(y-2)$$

7. Verify that

$$\begin{pmatrix} -e^{2t} \\ e^{2t} \end{pmatrix}$$

and

$$\begin{pmatrix} te^{2t} \\ (1-t)e^{2t} \end{pmatrix}$$

are solutions of

$$\begin{aligned} x' &= 3x + y \\ y' &= -x + y \end{aligned}$$

Verify that these are linearly independent solutions by computing an appropriate determinant.

Write the general solution to this system of equations in the format

$$\begin{aligned} x(t) &= \dots \\ y(t) &= \dots \end{aligned}$$

Find the solution to this equation satisfying the initial condition

$$x(0) = 1; y(0) = 2$$

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8. Solve the system of linear differential equations with constant coefficients using the method of eigenvectors and eigenvalues. Show hand calculations for the eigenvalues and eigenvectors.

   If the eigenvalues are complex, you need to give real solutions to the equations.

$$x' = 2x + y$$
$$y' = -x + 2y$$

   Give the general solution to this system of equations in the form

$$x(t) = \dots$$
$$y(t) = \dots$$

   (i.e., the final answer will not involve vector notation).