M 387 Assignment 1

Dr. Holmes

January 14, 2004

This was handed out Jan 14 and will be due Jan 21.

1. Give a formal justification of the following proposed derived rule:

1. $\neg Q$ hypothesis for proof by contrapositive. . . . n. $\neg P$???

   n+1. $P \rightarrow Q$ proof by contrapositive, lines 1-n.

   Your justification should be a correct formal proof containing the given subproof and ending with the line $P \rightarrow Q$ (look at the justification of "proof by contradiction" in the notes for an example of formal justification of a rule of this kind).

   You may use derived rules from the notes in your proof.

2. Give a formal proof of the deMorgan law $\neg(A \vee B) \leftrightarrow \neg A \wedge \neg B$. You may use derived rules listed in the notes in your proof.

3. I notice that I haven't included rules for the biconditional in my notes; I did talk about them in class. A simple way to deal with the biconditional (adequate for problem 2) is to use the rules

$$\frac{P \leftrightarrow Q}{(P \rightarrow Q) \wedge (Q \rightarrow P)}$$

   and

$$\frac{(P \rightarrow Q) \wedge (Q \rightarrow P)}{P \leftrightarrow Q}$$

Write down and justify a set of introduction and elimination rules for the biconditional closer to the style of the rules given for other connectives. I said something in class about this, so this shouldn't be hard. I suggest that you might want *four* elimination rules.

4. Fun with truth tables.

   (a) Verify my assertions that $\leftrightarrow$ and $\oplus$ are associative. Give English translations of $P \leftrightarrow Q \leftrightarrow R$ (hint: this does *not* mean that P, Q and R are equivalent!) and $P \oplus Q \oplus R$.

   (b) There are sixteen possible truth tables for operations on two propositions P and Q. List them, and for each operation give some kind of English description of its meaning and an expression for the operation in our notation.

   For example, one of the 16 possible truth tables is

P	Q	
T	T	T
T	F	T
F	T	F
F	F	F

   and the operation expressed by it can briefly be expressed as P.