Math 387, Spring 2000: Test One

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February 18, 2000

(this version incorporates some slight corrections given in class)

This exam is open book, open notes, closed neighbor. Part of the exam has already been handed out as a set of exercises to take home; I remind you that these are due on February 22nd at 5 pm. The exam lasts from 9:40 to 10:35.

One question will be dropped. Moreover, you will receive two copies of the exam; one of these can be taken home, and I will consider work turned in to me on Tuesday, February 22nd for partial credit. If you do take-home work on the second copy of your exam, the same rules apply as for the take-home part of the exam: do not consult with anyone but me.

A copy of the corrected problem 1 from the take-home is also attached. It is worth a bonus point (1 out of 10 for the problem) to explain why the original version with the typo is a silly question (it is too easy; to get the point you need to give the correct answer and a correct, brief argument for that answer).

1. Confirm or refute the assertion $A \wedge B \equiv A \leftrightarrow B \leftrightarrow (A \vee B)$ using truth tables.

2. Give a definition of $P \oplus Q$ using the connective NOR and no other propositional connective.

   P NOR Q is true iff P and Q are both false.

3. Convert the DNF formula $(P \wedge \neg Q) \vee (Q \wedge \neg R)$ to

   (a) CNF, with obvious simplifications carried out.

   (b) full DNF

   The two parts should be done separately; don't try to convert from full DNF to CNF or vice versa!

4. Describe the matches between variables and terms which result when one attempts to apply the given algebraic rule to the given term, or explain why matching fails, and give the result of the application of the algebraic rule to the term if it succeeds. No use of commutative or associative matching is assumed.

   For example, if one applies the rule

$$x + y = y + x$$

   to the term $(a + b) + c$, the variable x will match $a + b$, the variable y will match c, and the result of applying the rule will be $c + (a + b)$.

   (a) Apply the rule $A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$ to the formula $(S \vee T) \wedge (U \vee (V \rightarrow W))$.

   (b) Apply the rule $(x + y)z = xz + yz$ to the term $a(b + cd)$.

5. Annotate the given (correct) natural deduction proof, completing all Goal: statements and providing correct justifications for each numbered line.

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Goal: ((A -> C) | (B -> C)) |- ((A & B) -> C)
1. (A -> C) | (B -> C)
Goal:
| 2. A & B
|
| Goal:
| Goal:
| | 3. ~C
| | Goal:
| | | 4. A -> C
| | |
| | | 5. A
| | |
| | | 6. C
| | |
| | | 7. C & ~C
| | 8. ~(A -> C)
| |
| | 9. B -> C
| |
| | 10. B
| |
| | 11. C
| |
| | 12. C & ~C
|
| 13. ~~C
|
| 14. C

15. (A & B) -> C
```

Most credit rests in justifying each numbered line, but I would also like to see the Goal: lines filled in sensibly.

6. Prove $((A \wedge B) \vee (A \wedge \neg B)) \rightarrow A$ by natural deduction. Individual lines are to be justified by rules taken from my lecture notes or from Grantham's book.

7. Prove $(A \to B) \leftrightarrow (\neg A \vee B)$ by natural deduction. Individual lines are to be justified by rules taken from my lecture notes or from Grantham's book.