

# Notes for Test I Preparation

Dr. Holmes

September 23, 2004

## 1 General Principles

This is a closed book, closed notes test. Some information of my choosing may be supplied with your test paper.

Expect to be tested on whether you have read the assigned readings.

You will be asked to do or evaluate mathematical reasoning only with axioms and definitions available to you. (This doesn't apply to knowledge which falls under the prerequisites of the course – I might presume high school knowledge or very simple information which I presume you would have from Math 187 or Phil 201).

One question on the exam (which will count for a small number of points) will ask for a brief description of your first essay topic!

There will be an essay question. You will have a choice between a number of questions, and some possible topics are indicated here (some or all of the possibilities indicated here will appear as alternatives, so you can prepare your ideas in advance). I will not expect anything incredibly polished: the standard is intelligent presentation of a thesis and some argument in favor of it.

You are positively encouraged to ask questions in class to draw me out about what I may mean by some of the cryptic statements in this document!

## 2 Review of what we have covered

I might ask some questions about “prehistory” issues: review the reading on Egyptian and Babylonian mathematics. Be familiar with the systems of numeration of these two cultures in particular (you don't need to know their

symbols, just the basic structure of the numerals). This is not the only thing I might ask about, just the most obvious.

I might ask an essay question about the status of the objects of mathematics. What are we really talking about when we do mathematics? What is the relation of mathematical objects to the objects we are familiar with in the physical world? (This would be one of a menu of alternative essay questions). Here you are encouraged to think for yourself; ideas we have covered so far in class are not the only ones which can be applied.

On ancient Greek mathematics, we have covered the following topics: the Pythagoreans, and the crisis of their philosophy due to the discovery of the irrationality of the square root of 2. The paradoxes (including Zeno's paradoxes and more modern ones such as the paradox of the gods). The specific paradoxes we have talked about are the paradox as to whether one can go anywhere (first you have to go halfway...), the paradox of Achilles and the tortoise, and the paradox of the gods. A short answer question might ask for a description of one of these.

Another essay question might ask for a full description and discussion of a paradox of your choice (you may choose any paradox in the Moore reading). Your essay should indicate clearly where the paradoxical quality comes from and suggest some way (or some different ways) that the paradox could be resolved. What makes a "paradox" different from a plain contradiction in a mathematical system?

Notice that I mentioned another paradox (the least natural number not describable in English in less than one billion words); this is called "Richard's paradox" and is part of the later set of paradoxes of logic and set theory which popped up around the turn of the last century. It is described in the notes for Sept. 21.

The next topic is axiomatizations of geometry (Euclid and Hilbert). You may be asked to say something about differences between the approaches of Euclid and Hilbert (definitions are treated differently; the explicit recognition that some notions cannot be defined is modern, though it is analogous to the Greek recognition that some theorems cannot be proved). You should think about differences in their attitude toward their subject matter (there are historical and philosophical reasons which I or the readings may suggest for differences between Euclid and Hilbert).

I may present proofs from Euclid and ask what is missing (what assumption is Euclid using which is not found in his axioms?). I may present proofs from Euclid and/or Hilbert and ask you to justify a step in the reasoning. In

either case, you will have the text of the proof and the relevant axioms and definitions available with your test paper.

I wrote some notes about desirable properties of systems of axioms and definitions which are posted on the web. Recall that I gave examples of more and less economical sets of primitive notions using logical connectives: I regard this as a fair topic to ask questions about (a review of the propositional logic could be useful; I will supply truth tables for any connectives I talk about).

Some kind of essay question on values to consider in formulating primitive notions and axioms might materialize. (Russell's remarks on "mathematical philosophy" are relevant.) Short answer questions about the propositional logic example are to be expected.

Now we turn to the Russell reading.

You should understand what the Peano axioms say. You will not be called on to produce any proof at the level of the complicated definition of addition I talked about on Tuesday!!! I'll do an example or two Thursday of simple things you might be asked to do (show that two times two is four :-). I might write out some kind of proof and ask you to comment on its structure. In any event, the axioms and the definitions of addition and multiplication will be available.

You should understand the "structural" approach to understanding mathematical language and the difference between this approach and any proposed specific definition of the natural numbers. I'll be talking about this more today.

You should understand the Frege/Russell definition of the natural numbers (which will be introduced in class today). You might want to think about whether you like this definition, and what is good about it and what is bad about it. A possible essay question would be to write an explanation of this definition and explain why you think it is a good definition or a bad definition. I will explicitly talk about a major problem with this definition which Russell does not explicitly discuss in the book.

This is as far as the test will go. Again, you are encouraged to ask me questions about this document today and Tuesday; it will help me in my thinking about writing it! It does seem that there are plenty of things I can ask about, after writing it all down :-)