

# Assignment I

Dr. Holmes

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This assignment is due Thursday, June 10.

1. Let  $P$  represent “Snow is white”,  $Q$  represent “Grass is blue”, and  $R$  represent “It will rain”. Write out English sentences equivalent to each of the following expressions: make sure that your sentences really are grammatical English and are as clear as possible. Remember order of operations:  $\neg$  has the highest precedence, followed by  $\wedge$ ,  $\vee$ ,  $\rightarrow$  and  $\leftrightarrow$ . It doesn't hurt to add extra parentheses for clarity.

- (a)  $\neg Q$
- (b)  $\neg P \vee Q$
- (c)  $(P \rightarrow Q) \wedge (Q \rightarrow \neg R)$
- (d) \*  $(P \rightarrow Q) \rightarrow R$  The star means this part is harder. It might help to translate one or both of the implications into one of the logically equivalent forms we discussed in class.

Write out expressions of propositional logic equivalent to the following English sentences:

- (a) Either snow is not white or grass is not blue.
  - (b) If snow is white and grass is not blue, then it will rain.
  - (c) Snow is not white and if grass is not blue then it will rain.
2. Show that  $\neg(P \wedge \neg Q)$  is logically equivalent to  $P \rightarrow Q$ , using the method of truth tables.

3. Show that  $(P \wedge Q) \rightarrow R$  is logically equivalent to  $(P \rightarrow R) \vee (Q \rightarrow R)$ , using the method of truth tables.
4. Define a new binary connective (operation on propositions)  $P * Q$  (called the Sheffer stroke) to mean  $\neg(P \wedge Q)$ . Give a truth table for  $P * Q$ .  
 Show that  $\neg P$  is equivalent to  $P * P$  (easy). This implies that negation can be defined in terms of the Sheffer stroke. Show that  $P \wedge Q$  is equivalent to  $(P * Q) * (P * Q)$ ; this shows that conjunction can be defined in terms of the Sheffer stroke.  
 Show that the sentences  $P \vee Q$  and  $P \Rightarrow Q$  are logically equivalent to sentences which use just the Sheffer stroke.
5. There are 16 possible binary operations on propositions (think of all the possible four line truth tables). List the truth tables for all these operations and show that all of them can be expressed in terms of  $\wedge$ ,  $\vee$ , and  $\neg$ . Explain why this means that all of them can be expressed in terms of the Sheffer stroke.