Math 187 Assignment V

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This assignment is due Thursday, June 24.

1. Prove by mathematical induction that $n(n+1)(2n+1)$ is divisible by 6 for any natural number n.

   You may get additional credit if you can present a convincing argument *not* using mathematical induction for this assertion: you may use the fact that any natural number n is of one of the forms $3m$, $3m-1$ or $3m-2$, depending on its remainder on division by 3.

   (It is obvious from the formula for sums of squares given in the book that this must be true, but your proofs should not appeal to this.)

2. Prove by mathematical induction that

   $$\Sigma_{i=1}^{n} 2i - 1 = n^2$$

   .

   You can get additional credit by finding $f(i)$ such that

   $$\Sigma_{i=1}^{n} f(i) = i^3.$$

   Hint: use the telescoping sum property. But you might need to do some additional fiddling to get things right.

3. Give a complete proof for exercise 4.6.4. in the book. Use summation notation to state the result (the book uses dots) and use properties of summation notation appropriately in your proof.

4. Examine the Fibonacci sequence

$$1, 1, 2, 3, 5, 8 \ldots$$

Determine which of them are odd and which are even. When you have found a pattern, prove by mathematical induction that your pattern is correct. Hint: your statement might talk about more than one successive Fibonacci number, or you might need to use one of the modified forms of induction that we have discussed.

5. Prove by induction (the form is up to you, but you should have some idea what is likely to work with Fibonacci numbers) that $(f_{n-1})(f_{n+1}) = f_n^2 + (-1)^n$, for each $n \geq 2$ (where f_n is the nth Fibonacci number, as usual).

   Remember that you can express any f_k (with $k > 2$) in the form $f_{k-1} + f_{k-2}$.

6. Exercises on recursive definitions:

   The "Lucas numbers" are defined recursively by $L_1 = 1; L_2 = 3; L_{n+2} = L_n + L_{n+1}$. List the first ten Lucas numbers.

   Define a function recursively as follows:

   $g(1) = 100$;

   if $g(n)$ is odd, then $g(n+1) = 3n + 1$;

   if $g(n)$ is even, then $g(n+1) = \frac{g(n)}{2}$

   Find the smallest value of n such that $g(n) = 1$ (there is one); show all computations (so you will list all values of g for earlier values of n).