Math 187 Assignment V

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This assignment is due on Tuesday, June 29th. This version corrects an error in problem 1 in the original online version.

1. Let $f : \mathcal{R} \to \mathcal{R}$ and $g : \mathcal{R} \to \mathcal{R}$ be defined by $f(n) = 3n + 2$ and $g(n) = n^2 + 4$. $\mathcal{R}$ is the set of real numbers. Present simplified formulas for the compositions $f \circ g$ and $g \circ f$.

   One of these functions has an inverse and one doesn't; give the usual Math 147 style derivation of the inverse of the one and explain why the other does not have an inverse.

   Give a description of a function from the natural numbers to the natural numbers which is not the identity function but which is its own inverse.

2. Prove that if f is a surjection from A to B and g is a bijection from B to C then $g \circ f$ is a surjection from A to C.

3. Construct an example of functions $f : A \to B$ and $g : B \to A$ such that f is an injection, g is not an injection, but gf is still an injection. You can use finite sets A and B (arrow diagrams are acceptable). Is this possible if f is a bijection? If it is possible, give an example. If it is not possible, prove that it is not possible.

4. Let $A = \{1, 2\}$ and let $B = \{a, b, c\}$. How many functions are there from A to A? From A to B? From B to A? From B to B?

   Draw tree diagrams to illustrate the reasons for your answers.

   State a general rule for the number of functions from a set with m elements to a set with n elements.

5. Let $A = \{1, 2\}$, $B = \{a, b, c\}$ and $C = \{d, e, f, g\}$,

   How many injections are there from A to B? From A to C? From B to C?

   Draw tree diagrams to illustrate the reasons for your answers.

   How many injections are there from C to B? What theorem can be applied to give this answer easily?

6. Let A, B, and C be as in the previous problem. How many bijections are there from A to A? From B to B? From C to C?

   Draw tree diagrams to illustrate the reasons for your answers. Use the tree diagram for the case of bijections from C to C to list all the bijections systematically (you may use arrow diagrams or sets of ordered pairs to write down each bijection).

7. How many *surjections* are there from a set with 3 elements onto a set with 2 elements? From a set with 4 elements to a set with 2 elements?

   Can you give a general rule for the number of surjections from a set with n elements to a set with 2 elements?